

meaning, $x = \pm 0.5$ cm, $\theta = \pm 2.0$ mr, $y = \pm 1.3$ cm, $\phi = \pm 2.5$ mr, $l = \pm 0.0$ cm, $\delta = \pm 1.5$ percent $\Delta p/p$, and the central momentum $p(0) = 10.0$ GeV/c.

The units of the tabulated matrix elements in either the first-order R or sigma matrix or second order T matrix of a TRANSPORT print-out will correspond to the units chosen for the BEAM card. For the above example, the $R(12) = (x/\theta)$ matrix element will have the dimensions of cm/mr; and the $T(236) = (\theta/y\delta)$ matrix element will have the dimensions mr/(cm \cdot percent) and so forth.

The longitudinal extent l is useful for pulsed beams. It indicates the spread in length of particles in a pulse. It does not interact with any other component and may be set to zero if the pulse length is not important.

The phase ellipse (sigma matrix) beam parameters may be printed as output after every physical element if activated by a (13. 3. ;) element. Alternatively, individual printouts may be activated by a (13. 1. ;) element. The projection of the semi-axes of the ellipsoid upon each of its six coordinates axes is printed in a vertical array, and the correlations among these components indicating the phase ellipse orientations are printed in a triangular array (see the following pages).

The phase ellipse beam matrix

The beam matrix carried in the computer has the following construction:

	x	θ	y	φ	ℓ	δ
x	σ(11)					
θ	σ(21)	σ(22)				
y	σ(31)	σ(32)	σ(33)			
φ	σ(41)	σ(42)	σ(43)	σ(44)		
ℓ	σ(51)	σ(52)	σ(53)	σ(54)	σ(55)	
δ	σ(61)	σ(62)	σ(63)	σ(64)	σ(65)	σ(66)

The matrix is symmetric so that only a triangle of elements is needed.

In the printed output this matrix has a somewhat different format for ease of interpretation:

		x	θ	y	φ	ℓ
x	$\sqrt{\sigma(11)}$	CM				
θ	$\sqrt{\sigma(22)}$	MR	r(21)			
y	$\sqrt{\sigma(33)}$	CM	r(31)	r(32)		
φ	$\sqrt{\sigma(44)}$	MR	r(41)	r(42)	r(43)	
ℓ	$\sqrt{\sigma(55)}$	CM	r(51)	r(52)	r(53)	r(54)
δ	$\sqrt{\sigma(66)}$	PC	r(61)	r(62)	r(63)	r(64) r(65)

where:
$$r(ij) = \frac{\sigma(ij)}{[\sigma(ii)\sigma(jj)]^{\frac{1}{2}}} .$$

As a result of the fact that the σ matrix is positive definite, the r(ij) satisfy the relation

$$|r(ij)| \leq 1 .$$

The full significance of the $\sigma(ij)$ and the $r(ij)$ are discussed in detail in the Appendix ("Description of Beam Matrix"). The units are always printed with the matrix.

In brief, the meaning of the $\sqrt{\sigma(ii)}$ is as follows:

$\sqrt{\sigma(11)} = x_{\max}$ = the maximum (half)-width of the beam envelope in the x(bend)-plane at the point of the print-out.

$\sqrt{\sigma(22)} = \theta_{\max}$ = the maximum (half)-angular divergence of the beam envelope in the x(bend) plane.

$\sqrt{\sigma(33)} = y_{\max}$ = the maximum (half)-height of the beam envelope.

$\sqrt{\sigma(44)} = \phi_{\max}$ = the maximum (half)-angular divergence of the beam envelope in the y(non-bend)-plane.

$\sqrt{\sigma(55)} = \ell_{\max}$ = one-half the longitudinal extent of the bunch of particles.

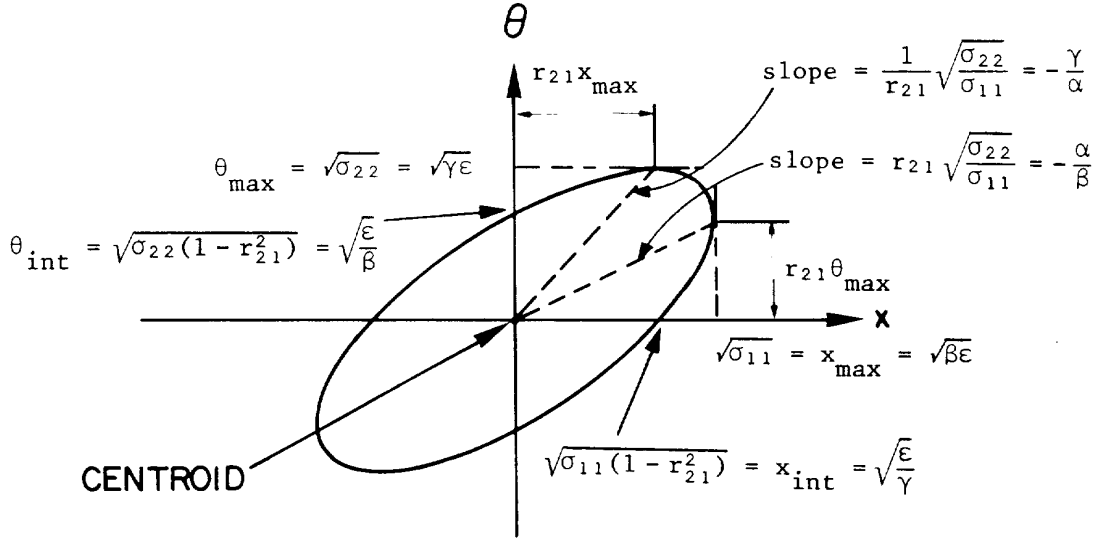
$\sqrt{\sigma(66)} = \delta$ = the half-width ($\frac{1}{2} \Delta p/p$) of the momentum interval being transmitted by the system.

The units appearing next to the $\sqrt{\sigma(ii)}$ in the TRANSPORT print-out are the units chosen for coordinates x , θ , y , ϕ , ℓ and $\delta = \Delta p/p$, respectively.

To the immediate left of the listing of the beam envelope size in a TRANSPORT print-out, there appears a column of numbers whose values will normally be zero. These numbers are the coordinates of the centroid of the beam phase ellipse (with respect to the initially assumed central trajectory of the system). They may become non-zero under one of three circumstances:

- 1) when the misalignment (type code 8.0) is used,
- 2) when a beam centroid shift (type code 7.0) is used, or
- 3) when a second-order calculation (type code 17.0) is used.

To aid in the interpretation of the phase ellipse parameters listed above, an example of an (x, θ) plane ellipse is illustrated below. For further details the reader should refer to the Appendix of this report.



A TWO-DIMENSIONAL BEAM PHASE ELLIPSE

The area of the ellipse is given by:

$$A = \pi(\det \sigma)^{\frac{1}{2}} = \pi x_{\max} \theta_{\text{int}} = \pi x_{\text{int}} \theta_{\max}$$

The equation of the ellipse is:

$$\gamma x^2 + 2\alpha x\theta + \beta \theta^2 = \epsilon$$

where

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

and

$$\beta\gamma - \alpha^2 = 1, \quad r_{21} = r_{12} = \frac{-\alpha}{\sqrt{1+\alpha^2}} = \frac{-\alpha}{\sqrt{\beta\gamma}}$$

r.m.s. addition to the BEAM

To allow for physical phenomena such as multiple scattering, provision has been made in the program to permit an r.m.s. addition to the beam envelope. There are nine entries to be included:

- 1 - Type code 1.0 (specifying a BEAM entry follows).
- 2 - The r.m.s. addition to the horizontal beam extent (Δx) (cm).
- 3 - The r.m.s. addition to the horizontal beam divergence ($\Delta\theta$) (mr).
- 4 - The r.m.s. addition to the vertical beam extent (Δy) (cm).
- 5 - The r.m.s. addition to the vertical beam divergence ($\Delta\phi$) (mr).
- 6 - The r.m.s. longitudinal beam extent ($\Delta\ell$) (cm).
- 7 - The r.m.s. momentum spread ($\Delta\delta$) (in percent $\Delta p/p$).
- 8 - The momentum change in the central trajectory [$\Delta p(0)$] in (GeV/c).
- 9 - The code digit 0. indicating an r.m.s. addition to the BEAM is being made.

The units for the r.m.s. addition are the same as those selected for a regular BEAM type code 1.0 entry. Thus a typical r.m.s. addition to the BEAM would appear as follows:

1. .1 .2 .15 .3 0. .13 -0.1 0. ;

where the last entry (0.) preceding the semicolon signifies an r.m.s. addition to the BEAM is being made and the next to the last entry indicates a central momentum change of -0.1 GeV/c.

FRINGING FIELDS and POLE-FACE ROTATIONS for bending magnets:

Type code 2.0

To provide for fringing fields and/or pole-face rotations on bending magnets, the type code 2.0 element is used.

There are two parameters:

- 1 - Type code 2.0.
- 2 - Angle of pole-face rotation (degrees).

The type code 2.0 element must either immediately precede a bending magnet (type code 4.0) element (in which case it indicates an entrance fringing field and pole-face rotation) or immediately follow a type code 4.0 element (exit fringing field and pole-face rotation) with no other data entries between *). A positive sign of the angle on either entrance or exit pole-faces corresponds to a non-bend plane focusing action and bend plane defocusing action.

For example, a symmetrically oriented rectangular bending magnet whose total bend is 10 degrees would be represented by the three entries
2. 5. ; 4. --- ; 2. 5. ;

The angle of rotation may be varied. For example, the element
2.1 5. ; would allow the angle to vary from an initial guess of 5 degrees to a final value which would, say, satisfy a vertical focus constraint imposed upon the system. See the type code 10.0 section for a complete discussion of vary codes.


Even if the pole-face rotation angle is zero, 2. 0. ; entries must be included in the data set before and after a type code 4.0 entry if fringing-field effects are to be calculated.

A single type code 2.0 entry that follows one bending magnet and precedes another will be associated with the latter.

*) It is extremely important that no data entries be made between a type code 2.0 and a type code 4.0 entry. If this occurs, it may result in an incorrect matrix multiplication in the program and hence an incorrect physical answer. If this rule is violated, an error message will be printed.

Should it be desired to misalign such a magnet, an update element must be inserted immediately before the first type 2.0 code entry and the convention appropriate to misalignment of a set of elements applied, since, indeed, three separate transformations are involved. See section under type code 8.0 for a discussion of misalignment calculations and the section under type code 6.0 for a discussion of updates.

The type code signifying a rotated pole-face is 2.0. The input format is:

2. β . 'RO' ;  Label (if desired)

The units for β are degrees.

Pole-face rotation matrix

The first-order R matrix for a pole-face rotation used in a TRANSPORT calculation is as follows:

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\tan \beta}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tan(\beta-\psi)}{\rho_0} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Definitions: β = angle of rotation of pole face (see figure on following page for sign convention of β)

ρ_0 = bending radius of central trajectory

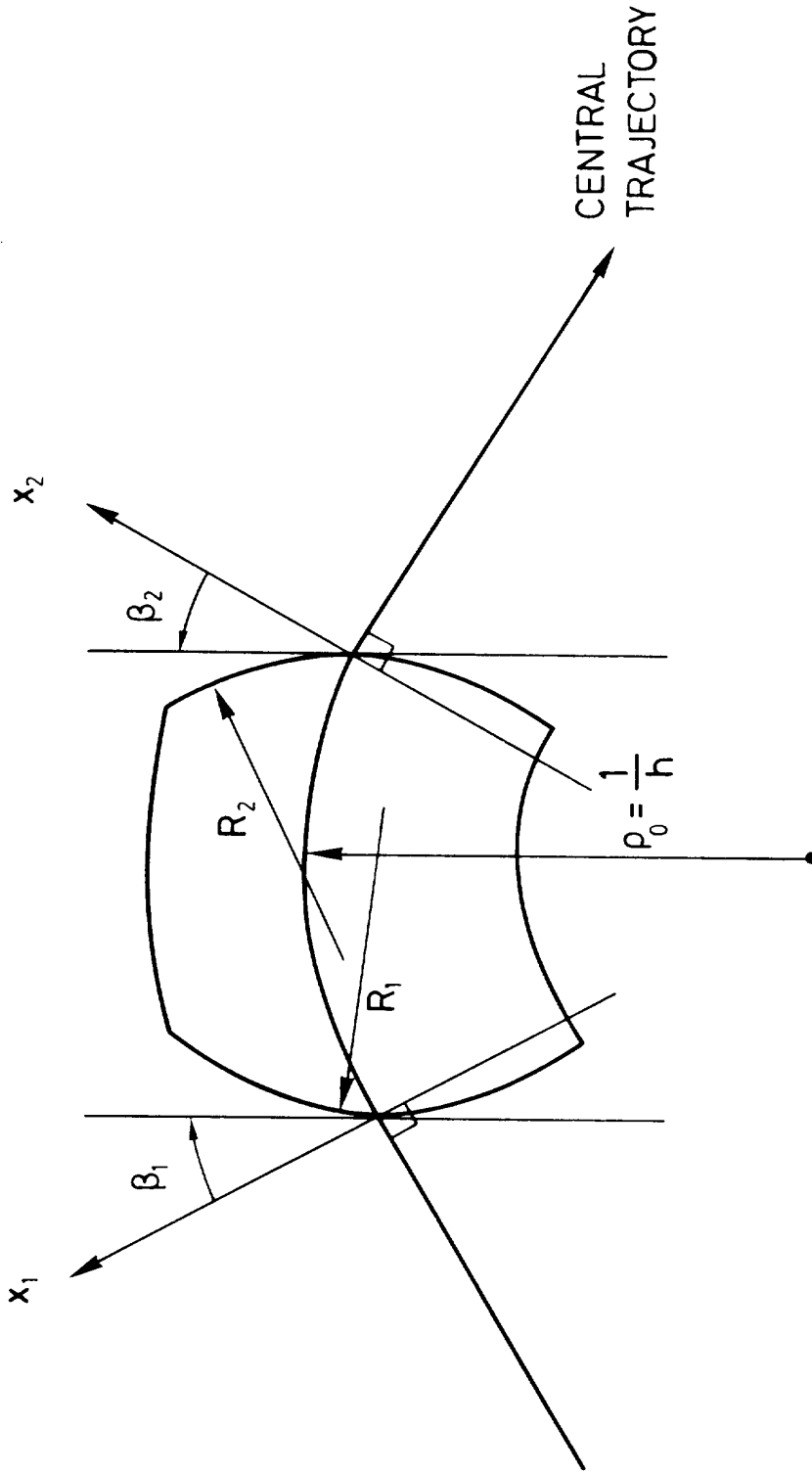
g = total gap of magnet

ψ = correction term resulting from spatial extent of fringing fields^{**}).

where
$$\psi = K_1 \left(\frac{g}{\rho_0} \right) \left(\frac{1 + \sin^2 \beta}{\cos \beta} \right) \left[1 - K_1 K_2 \left(\frac{g}{\rho_0} \right) \tan \beta \right]^*$$

^{*}) See type code 16.0 for input formats for g , K_1 , and K_2 TRANSPORT entries.

^{**}) See SLAC-75⁴) (page 74) for a discussion of ψ .



FIELD BOUNDARIES FOR BENDING MAGNETS

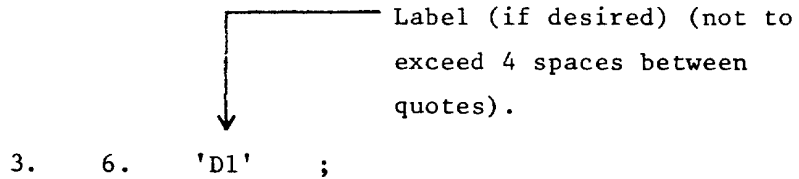
The TRANSPORT sign conventions for x , β , R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β 's imply transverse focusing. Positive R 's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R) \sec^3 \beta$. (See SLAC-75, page 71.)

DRIFT: Type code 3.0

A drift space is a field-free region through which the beam passes. There are two parameters:

- 1 - Type code 3.0 (specifying a drift length).
- 2 - (Effective) drift length (metres). The length of a drift space may be varied in either first- or second-order fitting.

Typical input format for a DRIFT:



DRIFT space matrix

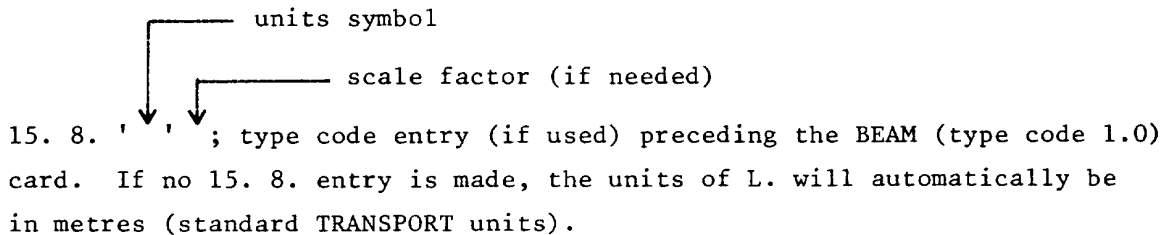
The first-order R matrix for a drift space is as follows:

$$\begin{pmatrix}
 1 & L & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & L & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1
 \end{pmatrix}$$

where

L = the length of the drift space.

The dimensions of L. are those chosen for longitudinal length via a



WEDGE BENDING MAGNET: Type code 4.0

A wedge bending magnet implies that the central trajectory of the beam enters and exits perpendicularly to the pole-face boundaries (to include fringing-field effects and non-perpendicular entrance or exit boundaries -- see type codes 2.0 and 16.0).

There are four first-order parameters to be specified for the wedge magnet via type code 4.0:

- 1 - Type code 4.0 (specifying a wedge bending magnet).
- 2 - The (effective) length L of the central trajectory in metres.
- 3 - The central field strength B(0) in kG,

$$B(0) = 33.356 (p/\rho_0),$$
 where p is the momentum in GeV/c and ρ_0 is the bending radius of the central trajectory in metres.
- 4 - The field gradient (n-value, dimensionless); where n is defined by the equation

$$B_y(x,0,t) = B_y(0,0,t) (1 - nhx + \dots),$$

where

$$h = 1/\rho_0. \text{ See SLAC-75 (page 31) }^4).$$

The quantities L, B(0), and n may be varied for first-order fitting (see type code 10.0 for a discussion of vary codes).

The bend angle in degrees and the bend radius in metres are printed in the output.

A typical first-order TRANSPORT input for a wedge magnet is

Label (not to exceed
4 spaces)

4. L. B. n. ' ' ;

If fringing field effects are to be included, a type code 2.0 entry must immediately precede and follow the pertinent type code 4.0 entry (even if there are no pole-face rotations). Thus a typical TRANSPORT input for a bending magnet including fringing fields might be:

Labels (not to exceed
4 spaces) if
desired

2. 0. ' ' ;
 4. L. B(0). n. ' ' ;
 2. 0. ' ' ;

For non-zero pole-face rotations a typical data input might be

```
2. 10. ; 4. L. B(0). n. ; 2. 20. ;
```

Note that the use of labels is optional and that all data entries may be made on one line if desired.

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the coordinate rotation matrix (type code 20.0) should be used as follows:

A bend up is represented by rotating the (x, y) coordinates by -90.0 degrees about the (z) beam axis as follows:

```

                Labels (not to exceed 4 spaces)
                if desired
20.  -90.  ' ' ;
2.   β(1).  ' ' ;
4.   L.   B.   n.   ' ' ;
2.   β(2).  ' ' ;
20.  +90.  ' ' ; (returns coordinates to their initial
                orientation)

```

A bend down is accomplished via:

```

20.  +90.  ' ' ;
2.
4.
2.
20.  -90.  ' ' ;

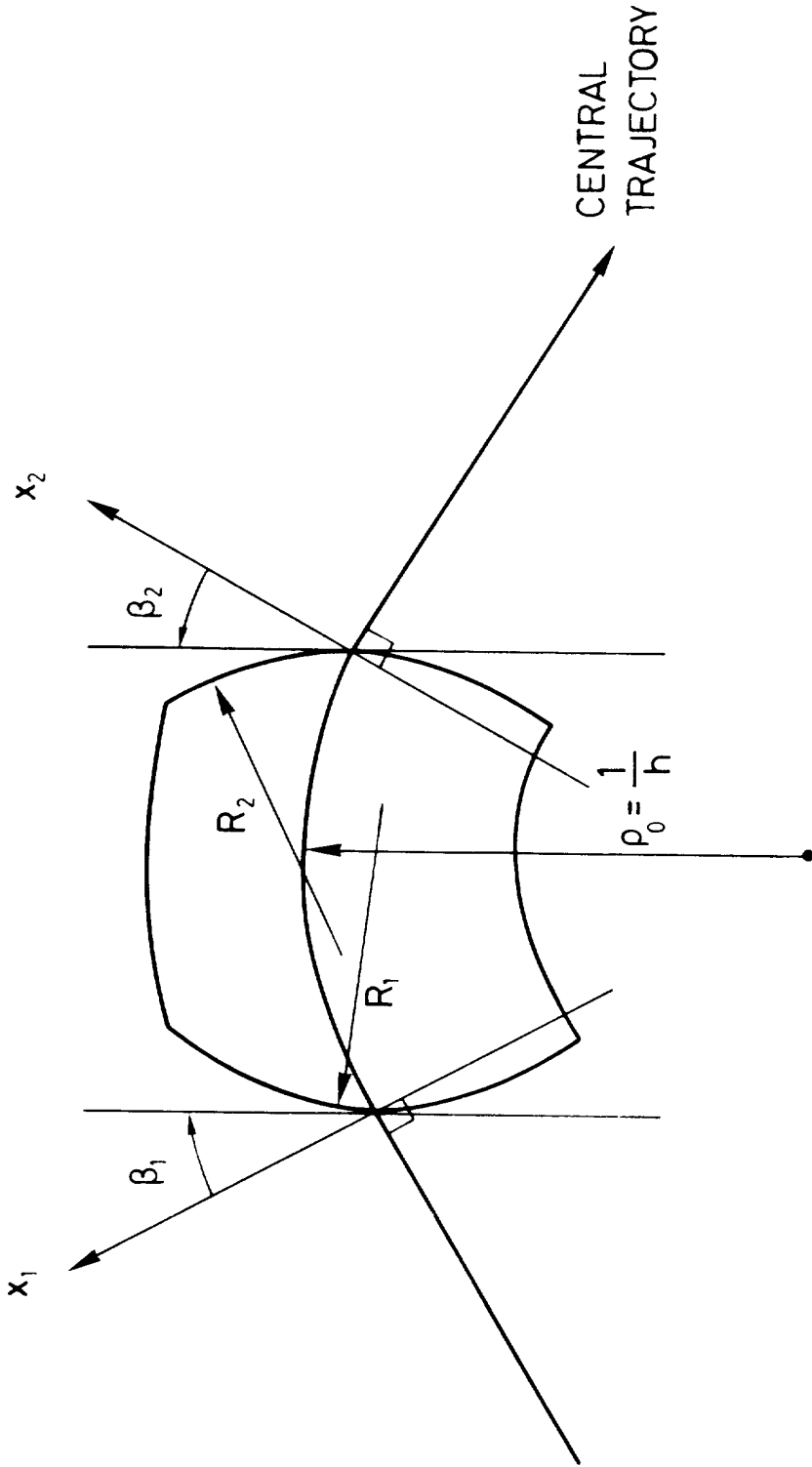
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x, y coordinates by 180 degrees, e.g.

```

20.  180.  ' ' ;
2.
4.
2.
20.  -180  ' ' ;

```



FIELD BOUNDARIES FOR BENDING MAGNETS

The TRANSPORT sign conventions for x , β , R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β 's imply transverse focusing. Positive R 's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R) \sec^3 \beta$. (See SLAC-75, page 71.)

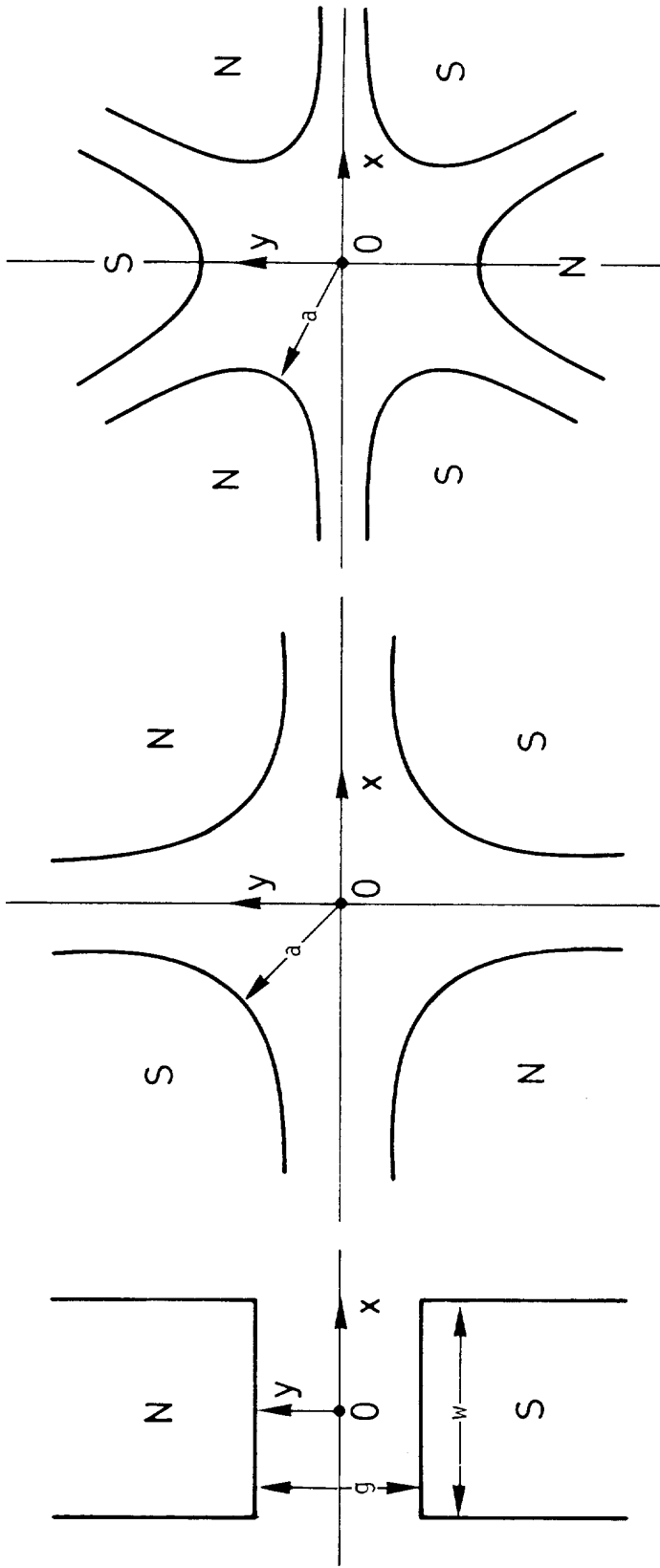
First-order wedge bending magnet matrix

$\cos k_x L$	$\frac{1}{k_x} \sin k_x L$	0	0	0	$\frac{h}{k_x^2} (1 - \cos k_x L)$
$-k_x \sin k_x L$	$\cos k_x L$	0	0	0	$\frac{h}{k_x} \sin k_x L$
0	0	$\cos k_y L$	$\frac{1}{k_y} \sin k_y L$	0	0
0	0	$-k_y \sin k_y L$	$\cos k_y L$	0	0
$-\frac{h}{k_x} \sin k_x L$	$-\frac{h}{k_x^2} (1 - \cos k_x L)$	0	0	1	$-\frac{h^2}{k_x^3} (k_x L - \sin k_x L)$
0	0	0	0	0	1

Definitions: $h = 1/\rho_0$, $k_x^2 = (1 - n)h^2$, $k_y^2 = nh^2$

$\alpha = hL$ = the angle of bend

L = path length of the central trajectory.



DIPOLE

QUADRUPOLE

SEXTUPOLE

ILLUSTRATION OF THE MAGNETIC MIDPLANE (x AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNET POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE WITH RESPECT TO EACH OTHER.

QUADRUPOLE: Type code 5.0

A quadrupole provides focusing in one transverse plane and defocusing in the other.

There are four parameters to be specified for a TRANSPORT calculation:

- 1 - Type code 5.0 (specifying a quadrupole).
- 2 - (Effective) magnet length L (in metres).
- 3 - Field at pole tip B (in kG). A positive field implies horizontal focusing; a negative field, vertical focusing.
- 4 - Half-aperture a (in cm). Radius of the circle tangent to the pole tips.

The length and field of a quadrupole may be varied in first-order fitting. The aperture may not be.

The strength of the quadrupole is computed from its field, aperture and length. The horizontal focal length is printed in parentheses as output. A positive focal length indicates horizontal focusing and a negative focal length indicates horizontal defocusing. The quantity actually printed is the reciprocal of the (θ/x) transfer matrix element $(1/R_{21})$ for the quadrupoles. Thus two identical quadrupoles of opposite polarity will have different horizontal focal lengths due to the difference between the sine and the hyperbolic sine.

The type code for a QUAD is 5.0. The input format for a typical data set is:

5. L. B. a. ' ' ;

Label (if desired) not to exceed
4 spaces between quotes

The standard TRANSPORT units for L, B, and a are metres, kG, and cm, respectively. If other units are desired they must be chosen via the appropriate 15.0 type code entries preceding the BEAM (type code 1.0) card.

First-order quadrupole matrix

$\cos k_q L$	$\frac{1}{k_q} \sin k_q L$	0	0	0	0
$-k_q \sin k_q L$	$\cos k_q L$	0	0	0	0
0	0	$\cosh k_q L$	$\frac{1}{k_q} \sinh k_q L$	0	0
0	0	$k_q \sinh k_q L$	$\cosh k_q L$	0	0
0	0	0	0	1	0
0	0	0	0	0	1

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

Definitions: L = the effective length of the quadrupole
a = the radius of the aperture
 B_0 = the field at radius a
 $k_q^2 = (B_0/a)(1/B\rho_0)$, where $(B\rho_0)$ = the magnetic rigidity (momentum) of the central trajectory.

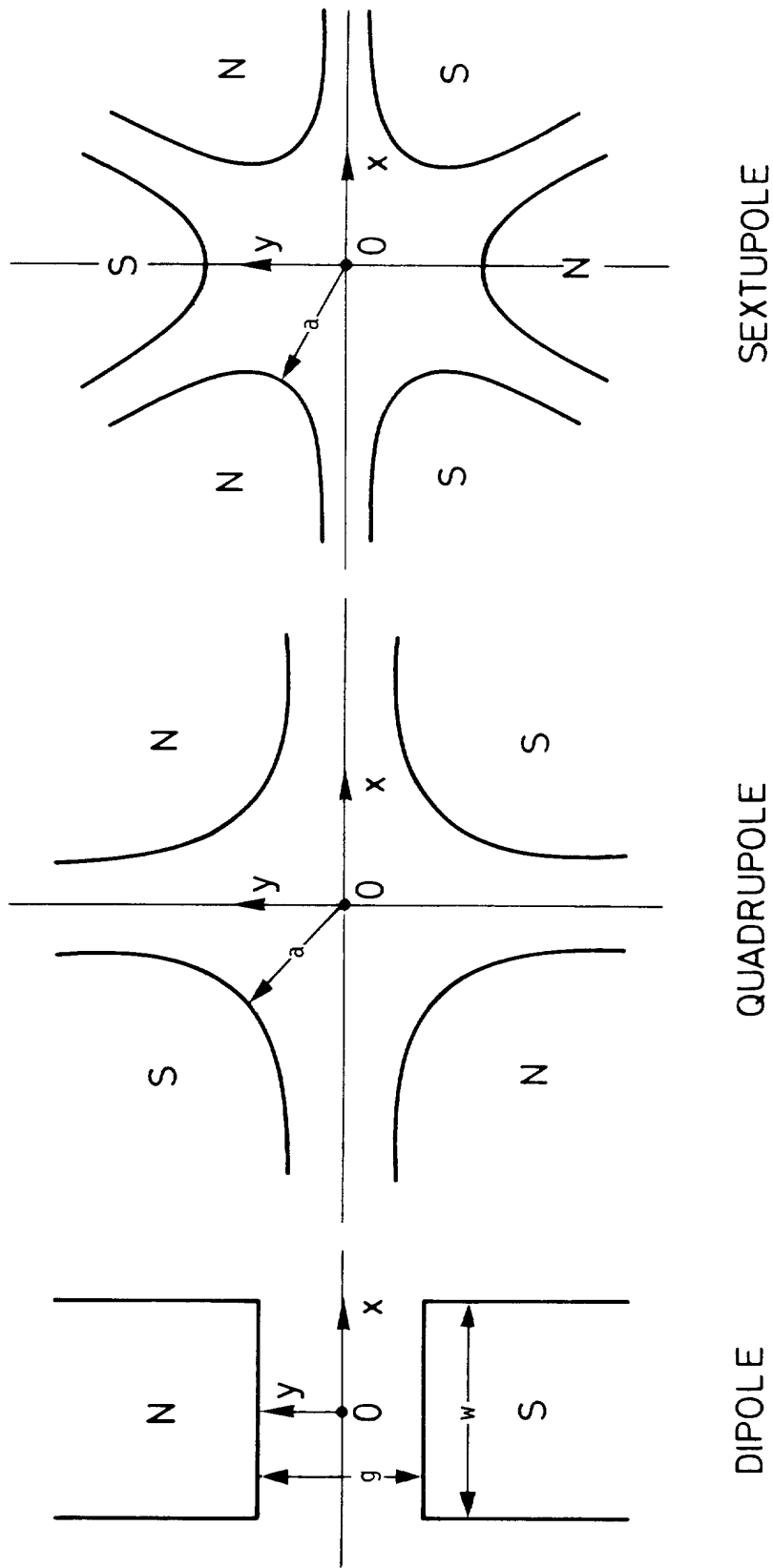


ILLUSTRATION OF THE MAGNETIC MIDPLANE (x AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNET POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE WITH RESPECT TO EACH OTHER.

TRANSFORM 1 update *): Type code 6. 0. 1.

To re-initialize the matrix TRANSFORM 1 (the product of the R matrices, R1) use type code 6.0. A (6. 0. 1. ;) card effects an update of the R1 matrix and initiates the accumulation of a new product matrix at the point of the update. This facility is often useful for misaligning a set of magnets or fitting only a portion of a system.

The matrix R1 is updated by no other element. It is not used in the calculation of the beam matrix. The beam matrix is calculated from the auxiliary transfer matrix R2 described on the next page.

A TRANSFORM 1 matrix will be printed at any position in the data set where a (13. 4. ;) entry is inserted.

See the following section for the introduction of an auxiliary transformation matrix R2 (TRANSFORM 2) to avoid the need for TRANSFORM 1 updates.

The (6. 0. 1. ;) card also causes an update of the R2 matrix.

*) By "updating" we mean initiating a new starting point for the accumulation (multiplication) of the R matrix. At the point of update the previous accumulation is discontinued. When the next element possessing a transfer matrix is encountered, the accumulated transfer matrix R1 is set equal to the individual transfer matrix R for that element. Accumulation is resumed thereafter.

AUXILIARY TRANSFORMATION MATRIX (R2): Type code 6. 0. 2.

The R1 matrix represents the accumulated transfer matrix from either the beginning of the beam line or the last explicit R1 update (6. 0. 1. ;). However several elements in TRANSPORT which affect the beam matrix cannot be represented in any transfer matrix. To avoid update complications with R1 an auxiliary transfer matrix R2 exists. The beam matrix is then calculated from the R2 matrix and the beam matrix at the last R2 update.

Both the R1 and R2 matrices are normally available for printing. However there is no redundancy in computer use, since, internally to the program only R2 is calculated at each element. The matrix R1 is calculated from R2 only as needed.

The R2 matrix is updated explicitly via a (6. 0. 2. ;) entry. It may be printed by a (13. 24. ;) entry. Constraints on R2 are imposed similarly to those on R1. For details see the section describing type code 10.0.

The complete list of elements which update TRANSFORM 2 is:

- 1) a beam type code 1.0 entry
- 2) the (6. 0. 1. ;) entry
- 3) the (6. 0. 2. ;) entry
- 4) a centroid shift type code 7.0 entry
- 5) a misalignment type code 8.0 entry
- 6) a stray field type code 21.0 entry.

Please note that automatic updates of TRANSFORM 2 occur when an align element (type code 8.) is inserted specifying the misalignment of all subsequent bending magnets. These TRANSFORM 2 updates take place immediately before and immediately after any bending magnet which has either the entrance or exit fringe fields specified via a type code 2 entry.

SHIFT IN THE BEAM CENTROID: Type code 7.0

Sometimes it is convenient to redefine the beam centroid^{**)} such that it does not coincide with the TRANSPORT reference trajectory. Provision has been made for this possibility via type code 7.0. Seven parameters are required:

- 1 - Type code 7.0.
- (2 to 7) - the coordinates x , θ , y , ϕ , l , and δ defining the shift in the location of the beam centroid with respect to its previous position. The units for x , θ , y , ϕ , l , δ are the same as those chosen for the BEAM (type code 1.0 entry), normally cm, mr, cm, mr, cm, and percent.

Any or all of the six beam centroid shift parameters may be varied in first-order fitting. The centroid position may then be constrained at any later point in the beam line by this procedure.

The transformation matrix R2 is updated by this element.

In order for this code to function properly, the initial BEAM entry (type code 1.0) must have a non-zero phase space volume, for example a

1. 0 0 0 0 0 0 p(0). ;

BEAM entry is not permissible when calculating a shift in the beam centroid; whereas a

1. 1. 1. 1. 1. 1. 1. p(0). ;

entry (non-zero phase volume) is acceptable.

^{**)} By "beam centroid" we mean the centre of the beam ellipsoid.

MAGNET ALIGNMENT TOLERANCES: Type code 8.0

The first-order effects of the misalignment of a magnet or group of magnets are a shift in the centroid of the beam and a change in the beam focusing characteristics. Two varieties of misalignment are commonly encountered: 1) the magnet is displaced and/or rotated by a known amount; or 2) the actual position of the magnet is uncertain within a given tolerance. TRANSPORT has the capability of simulating the misalignment of either single magnets or entire sections of a beam line. Any combination of the above alternatives may be simulated through the use of the "align" element. The results may be displayed in either the printed output of the beam (sigma) matrix or tabulated in a special misalignment table (described below).

There are eight parameters to be specified:

- 1 - Type code 8.0 (specifying a misalignment).
- 2 - The magnet displacement in the horizontal direction (cm).
- 3 - A rotation about the horizontal axis (mr).
- 4 - A displacement in the vertical direction (cm).
- 5 - A rotation about the vertical axis (mr).
- 6 - A displacement in the beam direction (cm).
- 7 - A rotation about the beam direction (mr).
- 8 - A three-digit code number (defined below) specifying the type of misalignment.

The three displacements and three rotations comprise the six degrees of freedom of a rigid body and are used as the six misalignment coordinates. The coordinate system employed is that to which the beam is referred at the point it enters the magnet. For example, a rotation of a bending magnet about the beam direction (parameter 7 above) is referred to the direction of the beam where it enters the magnet. The units employed are the standard TRANSPORT units shown above, unless redefined by type code 15. entries. If the units are changed, the units of the misalignment displacements are those determined by the 15. 1. type code entry; the units for the misalignment rotations are those determined by the 15. 2. type code entry.

The misalignment of any physical element or section of a beam line may be simulated. Misaligned sections of a beam line may be nested. A

beam line rotation (type code 20.) may be included in a misaligned section. Thus, for example, one can simulate the misalignment of magnets that bend vertically. The arbitrary matrix (type code 14.) may not be included in a misaligned section. A misalignment must never be included in a second-order run (type code 17.).

A misalignment element may indicate that a single magnet or section of the beam line is to be misaligned, or it may indicate that all subsequent magnets of a given type (quadrupoles and/or bending magnets) are to be misaligned. The type of misalignment is specified in the three-digit code number, and the location of the type code 8. align element depends on the type of misalignment.

If a misalignment pertains to a single magnet or a single section of the beam line, then the misalignment element (type code 8.) must directly follow that magnet or section of the beam line. If a misalignment element indicates that all subsequent magnets of a given type are to be misaligned, it must precede the first of such magnets. Further description of the available types of misalignment is given in the table below.

The results of the misalignment may be displayed in either the beam (sigma) matrix or in a misalignment table. If the results are displayed in the beam (sigma) matrix, then that matrix is altered by the effects of the misalignment. The effects of additional misalignments cause further alterations, so that at any point along the beam line, the beam (sigma) matrix will contain the combined effects of all previous misalignments.

The misalignment table can be used to show independently the effects on the beam matrix of a misalignment in each degree of freedom of each misaligned magnet. Each new misalignment to be entered in the table creates a new set of six duplicates of the beam matrix. Printed for each duplicate beam matrix are the centroid displacement and the beam half width in each of the six beam coordinates. Each of the six matrices shows the combined result of the undisturbed beam matrix and the effect of the misalignment in a single coordinate of a single magnet or section of the beam line. In a single TRANSPORT run the results of misaligning up to ten magnets or sections of the beam line may be included in the misalignment table. Further requests for entry in the misalignment table will be ignored. Examples of such a table and the input which generated it are shown below.

When the user specifies that the actual position of the magnet(s) is uncertain within a given tolerance, the printout will show a change in the beam (sigma) matrix resulting from the effects of the misalignment(s). Thus, if one wishes to determine the uncertainty in the beam centroid resulting from uncertainties in the positioning of the magnets, the initial beam dimensions should be set to zero, i.e. the beam card entry at the beginning of the system should appear as follows:

1. 0. 0. 0. 0. 0. 0. p(0).

If it is desired to know the effect of an uncertainty in position on the beam focusing characteristics, then a non-zero initial phase space must be specified. The printout will then show the envelope of all possible rays, including both the original beam and the effects of the misalignment.

If the misalignment is a known amount, it may affect the beam centroid as well as the beam dimensions. Therefore one should place on the BEAM card the actual dimensions of the beam entering the system. For a known misalignment, the program requires that the initial beam specified by type code 1 must be given a non-zero phase volume, to insure a correct printout.

An align element pertaining to a single magnet or section of the beam line updates the BEAM (sigma) matrix and the R2 matrix, but not the R1 matrix. A misalignment element which indicates misalignment of all subsequent magnets of a given type will update the BEAM (sigma) matrix and the R2 matrix before each bending magnet with fringe fields and after each misaligned magnet of any type.

The tolerances may be varied. Thus, type-vary code 8.111111 permits any of the six parameters (2 through 7 above) to be adjusted to satisfy whatever BEAM constraints may follow. For fitting, a misalignment must pertain to a single magnet or single section of the beam line, and the results must be displayed in the beam (sigma) matrix. (See the section under type code 10. for a discussion of the use of vary codes.)

The meaning of the options for each digit of the three-digit code number is given in the following table.

A. The units position specifies the magnet(s) or section of the beam line to be misaligned.

CODE NUMBER	INTERPRETATION
XX0.	The <u>single</u> magnet (type code element) immediately preceding the align card it to be misaligned. A bending magnet with fringe fields should be misaligned using one of the options described below.
XX1.	The last R1 matrix update (the start of the beam line or a 6. 0. 1. ; type code entry) marks the beginning of the section to be misaligned. The misalignment element itself marks the end. The section is treated as a unit and misaligned as a whole. The misalignments of quadrupole triplets and other combinations involving more than two quadrupoles may be studied using this code digit.
XX2.	<p>The last R2 matrix update (see type code 6. for a list of elements which update R2) marks the beginning of the misaligned section. The misalignment element marks the end. This option makes use of the fact that R2 matrix updates do not affect the R1 matrix.</p> <p>A bending magnet with fringing fields or pole face rotations (type code 2.) should be misaligned using this option. See examples 1 and 2 below for an illustration of this.</p> <p>An array of quadrupoles provides another example of the use of this option. By successive application of align elements, the elements of a quadrupole triplet could be misaligned relative to each other, and then the triplet as a whole could be misaligned. See example 3 below for an illustration of this.</p>
XX3.	All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified. This option is useful in conjunction with the tabular display of the misalignment results (see below). A bending magnet, with fringing fields included, is treated as a single unit and misaligned accordingly.

XX4. All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified. See XX3 above for further comments.

XX5. All subsequent quadrupoles are independently misaligned by the amount specified. See example 4 below for an illustration of this. See XX3 above for further comments.

B. The tens position defines the mode of display of the results of the misalignment.

X0X. The beam matrix contains the results of the misalignment. The beam matrix is printed wherever a 13. 1. ; card is encountered. The beam matrix will then contain contributions from all previous misalignments.

X1X. A table is used to store the results of misalignments. The effect of up to ten independently misaligned magnets may be shown in the table in a single run. The table is printed via a 13. 8. ; card, and may be compared with the undisturbed beam matrix (printed by a 13. 1. ; card) at any point. An example of such a table is shown below.

C. The hundreds position distinguishes between an uncertainty in position (0XX.) or a known displacement (1XX.).

Any combination of digits may be used to define the exact circumstances intended. Thus, code 111. indicates the deliberate displacement of a set of magnets (referred to the point where the beam enters the set), with the results to be stored in a table.

Example No. 1: A bending magnet with a known misalignment

A bending magnet (including fringe fields) misaligned by a known amount might be represented as follows:

```
3. L(1). ;  
6. 0. 2. ;  
2. 0. ; 4. 1. B. n. ; 2. 0. ;  
8. 0. 0. 0. 0. 0. 2. 102. ;  
3. L(2). ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the 6. 0. 2. ; transform 2 update and the misalignment code number XX2 is necessary because the magnetic array (bending magnet + fringing fields) consists of three type code elements instead of one.

Example No. 2: A bending magnet with an uncertain position

A bending magnet having an uncertainty of 2 mr in its angular positioning about the incoming beam (z axis) would be represented as follows:

```
3. L(1). ;
6. 0. 2. ;
2. 0. ; 4. L. B. n. ; 2. 0. ;
8. 0. 0. 0. 0. 0. 2.0 002. ;
3. L(2). ;
```

To observe the uncertainty in the location of the outgoing beam centroid, the input BEAM card should have zero phase space dimensions as follows:

```
1. 0. 0. 0. 0. 0. 0. p(0). ;
```

If the beam dimensions on the input BEAM card are non-zero, the resultant beam (sigma) matrix will show the envelope of possible rays, including both the input beam and the effect of the misalignment.

Example No. 3: A misaligned quadrupole triplet

One typical use of both the R1 and R2 matrices is to permit the misalignment of a triplet. For example, an uncertainty in the positions within the following triplet

```
5. 1.  -8. 10.  ;
5. 2.   7. 10.  ;
5. 1.  -8. 10.  ;
```

may be induced by appropriate 8. elements as noted:

```
6. 0.   1.  ;
5. 1.  -8. 10.  ;
6. 0.   2.  ;
5. 2.   7. 10.  ;
5. 1.  -8. 10.  ;
8. --- --- --- --- --- 000.  ;
8. --- --- --- --- --- 002.  ;
8. --- --- --- --- --- 001.  ;
```

The first 8. card in the list refers to the misalignment of the third magnet only. The second 8. card refers to the misalignment of the second and third magnets as a single unit via the R2 matrix update (the 6. 0. 2. ; entry). The last 8. card refers to the misalignment of the whole triplet as a single unit via the R1 matrix update (the 6. 0. 1. ; entry).

The comments about the BEAM card (type code 1. entry) in example 2 above are applicable here also.

Example No. 4: Misaligned quadrupoles in a triplet

Individual uncertainties in the positions of the quadrupoles in the triplet in example no. 3 above may be induced by a single misalignment as follows:

8. --- --- --- --- --- --- 015. ;
5. 1. -8. 10. ;
5. 2. 7. 10. ;
5. 1. -8. 10. ;

The effect of each misalignment coordinate on each quadrupole will be stored separately in a table. This table is printed wherever a 13. 8. ; type code is inserted.

