## REPETITION: Type code 9.0

Many systems include a set of elements that are repeated several times. To minimize the chore of input preparation, a 'repeat' facility is provided.

There are two parameters:

1 - Type code 9.0

2 - Code digit. If non-zero, it states the number of repetitions desired from the point it appears. If zero it marks the end of a repeating unit.

For example, a total bend of 12 degrees composed of four 3-degree bending magnets each separated by 0.5 metres could be represented by 9. 4. ; 4. --- ; 3. .5 ; 9. 0. ; Those elements (in this case a bend and drift) between the 9. 4. ; and 9. 0. ; would be employed four times.

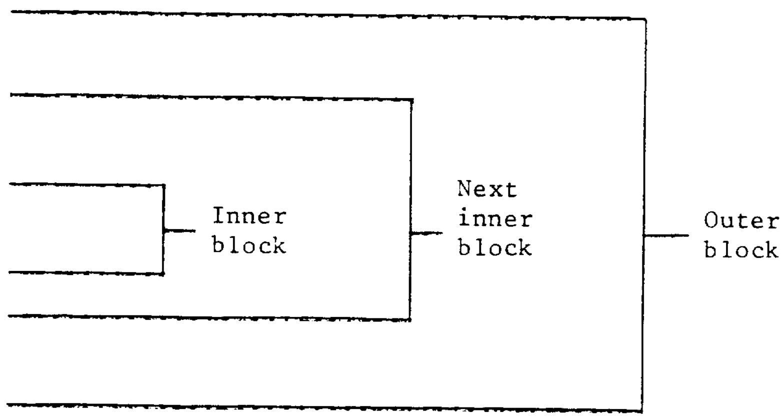
There is no indication of the 9.0 cards in the printed TRANSPORT output when calculating except for the repeated listing of the elements they control.

Vary codes may be used within a repeating unit in the usual fashion. However all repetitions of a given varied element will be coupled.

Repeat cards may be nested four deep. By "nesting" we mean a repeat within a repeat. An example is given below.

*Example of Nesting*

| | | |
|---|---|---|
| 9. | 2. | ; |
| 3. | 10. | ; |
| 9. | 3. | ; |
| 3. | 20. | ; |
| 9. | 4. | ; |
| 3. | 50. | ; |
| 9. | 0. | ; |
| 9. | 0. | ; |
| 3. | 1.5 | ; |
| 9. | 0. | ; |




The total length of this sequence is:

2*(10. + 3*(20 + 4* 50) + 1.5) = 1343.

## VARY CODES and FITTING CONSTRAINTS: Type code 10.0

Some (not all) of the physical parameters of the elements comprising a beam line may be varied in order to fit selected matrix elements. In a first-order calculation one might fit elements of the R1 or R2 transformation matrices or the beam (sigma) matrix. In second order one might constrain an element of the second-order matrix T1 or minimize the net contribution of aberrations to a given beam coordinate. Special constraints are also available.

One may not mix orders in fitting. First order vary codes and constraints must be inserted only in a first-order calculation, and similarly for second order.

The physical parameters to be varied are selected via 'Vary Codes' attached to the type codes of the elements comprising the system. The fitting constraints on matrix elements are selected via type code 10.0 entries placed in the system where the constraint is to be imposed.

### *Vary codes*

Associated with each physical element in a system is a vary code which specifies which physical parameters of the element may be varied. This code occupies the fraction portion of the type code specifying the element. It has one digit for each parameter, the digits having the same order in the code as the physical parameters have on the card. A '0' indicates the parameter may not be varied; a '1' that it may be. For instance, 3.0 is the combined type (3) and vary code (0) for a drift length which is to remain fixed; 3.1 indicates a drift length that may be varied (by the virtue of the .1). The type code 4.010 indicates a bending magnet with a variable magnetic field. In punching the code 3.0, the zero need not be punched. In punching the 4.010 code, the first zero must be punched but the second zero need not be.

*First-order vary codes*

In a first-order run the following parameters marked v may be varied, those marked 0 may not be varied.

| | |
|---|---|
| BEAM....... | 1.vvvvvv0 - All components of the input beam may be varied, except the momentum. |
| R.M.S. ADDITION.. | 1.vvvvvv00 - All components of an r.m.s. addition may be varied except the momentum change Δp. |
| ROTAT...... | 2.v - The pole face angle of a bending magnet may be varied. |
| DRIFT...... | 3.v - The drift length may be varied. |
| BEND....... | 4.vvv - The length, the field, and/or the n-value may be varied. |
| QUAD....... | 5.vv0 - The length may be varied; the field may be, the aperture may not be. |
| AXIS SHIFT.. | 7.vvvvvv - Any of the axis shift parameters may be varied. |
| ALIGN...... | 8.vvvvvv0 - Any of the alignment parameters may be varied. |
| INITIAL COORDINATES.. | 16.0v - Any of the three initial position floor coordinates or two angle coordinates may be varied. |
| MATRIX..... | 14.vvvvvv0 - Any of the first order matrix elements may be varied. |
| SOLENOID... | 19.vv - The length and/or field may be varied. |
| BEAM ROTATION.... | 20.v - The angle of rotation may be varied. |

The use of the permissive 'may' rather than the imperative 'will' in discussing variables is meaningful. The program will choose the parameters it will vary from among those that it may vary. In general it chooses to vary those parameters that have the greatest influence upon the conditions to be fit.

*Second-order vary codes*

In a second-order run the following parameters may be varied:

DRIFT..... 3.v - The drift length may be varied. Variation of a drift length should be done with caution as it may affect the first-order properties of the beam line. But inversely coupled drift spaces straddling a sextupole will, for example, show only second-order effects.

$\varepsilon(1)$...... 16.0v 1. - The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet may be varied.

1/R1...... 16.0v 12. - The pole face curvature of a bending magnet entrance may be varied.

1/R2...... 16.0v 13. - The pole face curvature of a bending magnet exit may be varied.

SEXTUPOLE 18.0v - The field strength may be varied.

The special parameter cards (type code 16.0) once introduced apply to all subsequent magnets in a beam line until another type code 16.0 specifying the same parameter is encountered. Thus if such a parameter is varied, the variation will apply simultaneously to all subsequent magnets to which it pertains. The variation will persist until the parameter or vary code attached to the parameter is changed by the introduction of another type code 16.0 card specifying the same parameter.

*Coupled vary codes*

It is possible to apply the same correction to each of several variables. This may be done by replacing the digit 1 in the vary code with one of the digits 2 through 9, or a letter A through Z. All such variables whose vary digits are the same, regardless of position will receive the same correction. For example, the three type-vary codes (5.0A, 5.01, 5.0A) might represent a symmetric triplet. The same correction will be made to the first and third quadrupoles, guaranteeing that the triplet will remain symmetric.

If a vary digit is immediately preceded by a minus sign, the computed correction will be subtracted from, rather than added to, this variable. Thus parameters with the same vary digit, one of them being preceded by a minus sign, will be inversely coupled. For example the type-vary code sequence (3.B, 5.01, 3.-B) will allow the quadrupole to move without altering the total system length.

Vary digits may also be immediately preceded by a plus sign without changing their meaning. Thus 5.0A is the same as 5.0+A. For historical reasons, the vary digits (9 and 4), (8 and 3), and (7 and 2) are also inversely coupled. Inverse coupling may not be used with type codes 1.0 or 8.0.

The total number of independent variables in a first-order run is limited to 20 by reasons of the mathematical method of fitting and to 10 for a second-order run. So far as this limit is concerned, variables that are tied together count as one. Variables within repeat elements (type code 9.0) also count only one.

*Possible fitting constraints*

A variety of possible constraints is available. Fitting may be done in either first- or second-order, but not in both simultaneously. The order of the constraint must be appropriate to the order of the run. A list of constraints available is given below. They are explained more fully on later pages.

First-order constraints

1) An element of the first-order transfer matrix R1.
2) An element of the auxiliary first-order matrix R2.
3) A $\sigma$ (BEAM) matrix element.
4) The correlations r in the beam coordinates.
5) The first moments of the beam.
6) The total system length.
7) An AGS machine constraint.
8) The reference trajectory floor coordinates.

Second-order constraints

1) An element of the second-order transfer matrix T1.
2) An element of the second-order auxiliary transfer matrix T2.
3) The net contributions of aberrations to a given coordinate of the beam matrix $\sigma$.
4) The strength of sextupoles used in the system.

The second-order matrices are actually computed using the auxiliary matrix T2. Therefore, when activating second-order fitting, one must not include any element which causes an update of the R2 matrix. For a complete list of such elements see type code 6.0.

The present value of the constrained quantity, as well as the desired value, is printed in the output. In the case of transfer matrix elements this value may be checked by printing the transfer matrix itself. Certain other constrained quantities may be checked similarly. Exceptions are noted in the explanations following.

<u>Rl matrix fitting constraints</u>

There are five parameters to be specified when imposing a constraint upon the (i, j) element of an Rl matrix.

1 - <u>Type code 10.</u> n(specifying that a fitting constraint follows).

2 - Code digit (-i).

3 - Code digit (j).

4 - Desired value of the (i, j) matrix element.

5 - Desired accuracy of fit (standard deviation).

Note that any fitting constraint on an Rl matrix element is from the preceding update of the Rl matrix. An Rl matrix is updated only by a (6. 0. 1. ;) entry.

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit.

Some typical R1 matrix constraints are as follows:

| Desired optical condition | Typical fitting constraint |
|---|---|
| Point to point imaging: | |
| Horizontal plane R(12) = 0 | 10. -1. 2. 0. .0001 'F1'; |
| Vertical plane R(34) = 0 | 10. -3. 4. 0. .0001 'F2'; |
| Parallel to point focus: | |
| Horizontal plane R(11) = 0 | 10. -1. 1. 0. .0001 'F3'; |
| Vertical plane R(33) = 0 | 10. -3. 3. 0. .0001 'F4'; |
| Point to parallel transformation: | |
| Horizontal plane R(22) = 0 | 10. -2. 2. 0. .0001 'F5'; |
| Vertical plane R(44) = 0 | 10. -4. 4. 0. .0001 'F6'; |
| Achromatic beam: | |
| Horizontal plane | 10. -1. 6. 0. .0001 'F7'; |
| R(16) = R(26) = 0 | 10. -2. 6. 0. .0001 'F8'; |
| Zero dispersion beam: | |
| Horizontal plane R(16) = 0 | 10. -1. 6. 0. .0001 'F9'; |
| Simultaneous point to point and waist to waist imaging: | |
| Horizontal plane | 10. -1. 2. 0. .0001 'F10' ; |
| R(12) = R(21) = 0 | 10. -2. 1. 0. .0001 'F11'; |
| Vertical plane | 10. -3. 4. 0. .0001 'F12'; |
| R(34) = R(43) = 0 | 10. -4. 3. 0. .0001 'F13'; |
| Simultaneous parallel to point and waist to waist transformation: | |
| Horizontal plane | 10. -1. 1. 0. .0001 'F14'; |
| R(11) = R(22) = 0 | 10. -2. 2. 0. .0001 'F15'; |
| Vertical plane | 10. -3. 3. 0. .0001 'F16'; |
| R(33) = R(44) = 0 | 10. -4. 4. 0. .0001 'F17'; |

## R2 matrix fitting constraints

There are five parameters to be specified when imposing a constraint upon the (i, j) element of an R2 matrix.

1 - Type code 10.n
2 - Code digit -(20 + i).
3 - Code digit (j).
4 - Desired value of the (i, j) matrix element.
5 - Desired accuracy of fit (standard deviation).

Some typical R2 matrix constraints are as follows:

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit.

| Desired optical condition | Typical fitting constraint |
|---|---|
| Point to point imaging: | |
| Horizontal plane R(12) = 0 | 10. -21. 2. 0. .001 'F1' ; |
| Vertical plane R(34) = 0 | 10. -23. 4. 0. .001 'F2' ; |
| Parallel to point focus: | |
| Horizontal plane R(11) = 0 | 10. -21. 1. 0. .001 'F1' ; |
| Vertical plane R(33) = 0 | 10. -23. 3. 0. .001 'F2' ; |
| Achromatic beam: | |
| Horizontal plane | 10. -21. 6. 0. .001 'F3' ; |
| R(16) = R(26) = 0 | 10. -22. 6. 0. .001 'F4' ; |

See type code 6.0 for a complete list of elements which update the R2 matrix.

σ(BEAM) matrix fitting constraints

There are five parameters to be specified when imposing a constraint upon the (i, j) element of a σ(BEAM) matrix.

1 - Type code 10.n
2 - Code digit (i). (i ≥ j)
3 - Code digit (j).
4 - Desired value of the (i, j) matrix element.
5 - Desired accuracy of fit (standard deviation).

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit. If i = j, then the value inserted in entry 4 is the desired beam size $(\sigma(ii))^{\frac{1}{2}}$ e.g. $x(max) = (\sigma(11))^{\frac{1}{2}}$ etc.

Some typical σ matrix constraints are as follows:

| Desired optical condition | Typical fitting constraint |
| --- | --- |
| Horizontal waist σ(21) = 0 | 10. 2. 1. 0. .001 'F1' ; |
| Vertical waist σ(43) = 0 | 10. 4. 3. 0. .001 'F2' ; |
| Fit beam size to x(max) = 1 cm | 10. 1. 1. 1. .001 'F3' ; |
| Fit beam size to y(max) = 2 cm | 10. 3. 3. 2. .001 'F4' ; |
| Limit max beam size to x = 2 cm | 10.2 1. 1. 2. .01 'F5' ; |
| Limit min beam size to y = 1 cm | 10.1 3. 3. 1. .01 'F6' ; |

In general, it will be found that achieving a satisfactory 'beam' fit with TRANSPORT is more difficult than achieving an R matrix fit. When difficulties are encountered, it is suggested that the user 'help' the program by employing sequential (step by step) fitting procedures when setting up the data for his problem. More often than not a "failure to fit" is caused by the user requesting the program to find a physically unrealizable solution. An often encountered example is a violation of Liouville's theorem.

Beam correlation matrix (r) fitting constraints

Five parameters are needed for a constraint on the (i, j) element of the beam correlation matrix.

1 - Type code 10.n
2 - Code digit (10 + i).
3 - Code digit (j).
4 - Desired value of the (i, j) matrix element.
5 - Desired accuracy of fit (standard deviation).

TRANSPORT does not print the beam ($\sigma$) matrix directly. Instead it prints the beam half widths and represents the off-diagonal elements by the correlation matrix. If one wishes to fit an element of this matrix to a non-zero value it is convenient to be able to constrain the matrix element directly.

Some typical r matrix constraints are as follows:

| Desired optical condition | Typical fitting constraint |
|---|---|
| Horizontal waist r(21) = 0 | 10. 12. 1. 0. .001 'F1' ; |
| yy' correlation = r(34) = 0.2 | 10. 13. 4. 0.2 .001 'F2' ; |

First moment constraint

In first order, known misalignments and centroid shifts cause the centre (centroid) of the phase ellipsoid to be shifted from the reference trajectory, i.e., they cause the beam to have a non-zero first moment. The first moments appear in a vertical array to the left of the vertical array giving the $\sqrt{\sigma(ii)}$. The units of the corresponding quantities are the same.

It is perhaps helpful to emphasize that the origin always lies on the reference trajectory. First moments refer to this origin. However, the ellipsoid is defined with respect to its centre, so the covariance matrix, as printed, defines the second moment about the mean.

First moments may be fitted. The code digits are i = 0 and j, where j is the index of the quantity being fit. Thus 10. 0. 1. .1 .01; constrains the horizontal (1.) displacement of the ellipsoid to be 0.1 ± 0.01 cm.

This constraint is useful in deriving the alignment tolerances of a system or in warning the system designer to offset the element in order to accommodate a centroid shift.

System length constraint

A running total of the lengths of the various elements encountered is kept by the program and may be fit. The code digits are i = 0., j = 0.

Thus the element (10. 0. 0. 150. 5. ;) would make the length of the system prior to this element equal to 150 ± 5 metres. Presumably there would be a variable drift length somewhere in the system. By redefining the cumulative length via the (16. 6. L. ;) element, partial system lengths may be accumulated and fit.

AGS machine constraint*)

Provision has been made in the program for fitting the betatron phase shift angle $\mu$, associated with the usual AGS treatment of magnet systems.

In the horizontal plane: use code digits i = -11., j = 2., and specify:

$$\Delta = \frac{1}{2\pi} \cos^{-1} \left[ 0.5 \ (R_{11} + R_{22}) \right] = \frac{\mu}{2\pi} \ \text{(horiz)}$$

$$= \text{freq./(No. of periods)}.$$

In the vertical plane: i = -13., j = 4., and

$$\Delta = \frac{1}{2\pi} \cos^{-1} \left[ 0.5 \ (R_{33} + R_{44}) \right] = \frac{\mu}{2\pi} \ \text{(vert)} \ .$$

For example, if there are 16 identical sectors to a proposed AGS machine and the betatron frequencies per revolution are to be 3.04 and 2.14 for the horizontal and vertical planes respectively, then the last element of the sector should be followed by the constraints:

```
10.  -11.  2.  .190  .001  ;
10.  -13.  4.  .134  .001  ;
```

$$\text{i.e.} \quad \frac{3.04}{16} = 0.190 \ \text{and} \ \frac{2.14}{16} = 0.134 \ .$$

For example: A typical data listing might be:

```
              5.01  ---   ;
              3.    ---   ;
              5.01  ---   ;
              3.    ---   ;

10.  -11.  2.  0.190  .001  ;
10.  -13.  4.  0.134  .001  ;
```

---

*) See Courant and Snyder[1]. Also note that this constraint is valid only when the unit cell structure and the corresponding beta functions are both periodic.

Floor coordinate fitting constraint

Five parameters are needed to specify a floor coordinate constraint:

1 - Type code 10.

2 - Code digit 8.

3 - Code digit (j).

4 - Desired value of floor coordinate.

5 - Desired accuracy of fit (standard deviation).

The code digit (j) indicates the floor coordinate to be constrained. Its possible values are 1 to 6 indicating the floor x, y, z, theta, phi, and psi, respectively. Theta is the angle which the floor projection of the reference trajectory makes with the floor z axis. Phi is the vertical pitch. Psi is a rotation about the reference trajectory. This is also the order in which coordinates are printed in the floor layout activated by the 13. 12. ; element. Initial coordinates are given on type codes 16. 16. ; through 16. 20. ; and type code 20.

The floor coordinates are actually zero-th rather than first order properties of a beam line. However, in TRANSPORT, they may be constrained in a first-order fitting run, and therefore are included here.

T1 matrix fitting constraints

Five parameters are needed for a constraint on the (i, j, k) element of the second-order transfer matrix T1.

1 - Type code 10.0
2 - Code digit (-i).
3 - Code digit (10j + k).
4 - Desired value of the (i, j, k) matrix element.
5 - Desired accuracy of the fit (standard deviation).

Note that upper and lower limit constraints are not available for second order fitting.

Some typical T1 matrix constraints are as follows:

| Desired optical condition | Typical fitting constraint |
|---|---|
| Geometric aberration $T_{122} = 0$ | 10. -1. 22. .0 .001 'F1' ; |
| Chromatic aberration $T_{346} = .5$ | 10. -3. 46. .5 .001 'F2' ; |

There must be no updates of the R2 matrix when constraining an element of the T1 matrix. There is no limit on the number of constraints which may be imposed.

If no drift lengths are varied the problem will be linear and the absolute size of the tolerances will be unimportant. Only their relative magnitude will be significant. Sometimes only a subset of the elements of the matrix $T_{ijk}$ which give significant contributions to beam dimensions need be eliminated. In such cases one may wish to minimize the effect of this subset, by weighing each matrix element according to its importance. One does this by including a constraint for each such matrix element, and setting its tolerance equal to the inverse of the phase space factor which the matrix element multiplies. For a matrix element $T_{ijk}$ acting on an uncorrelated initial phase space, the tolerance factor would be $1/(x_{0j}x_{0k})$, where $x_{0j}$ and $x_{0k}$ are the initial beam half widths specified by the type code 1.0 card.

T2 matrix fitting constraints

Five parameters are needed for a constraint on the (i, j, k) element of the second order auxiliary transfer matrix T2.

1 - Type code 10.0
2 - Code digit - (20 + i).
3 - Code digit (10j + k).
4 - Desired value of the (i, j, k) matrix element.
5 - Desired accuracy of the fit (standard deviation)

Note that upper and lower limit constraints are not available for second-order fitting.

Some typical T2 matrix constraints are as follows:

| Desired optical condition | Typical fitting constraint |
|---|---|
| Geometric aberration $T_{122} = 0$ | 10. -21. 22. .0 .001 'F1' ; |
| Chromatic aberration $T_{346} = .5$ | 10. -23. 46. .5 .001 'F2' ; |

By using a T2 constraint the user may fit an element of the second-order transfer matrix which pertains to any section of the beam. One causes an R2 update at the beginning of the section with a 6. 0. 2. ; element. One then places the T2 constraint at the end of the section. Any number of such constraints may be imposed. This is the only second-order constraint that may be used in conjunction with an R2 update.

If a printing of the T1 matrix is requested via a 13. 4. ; element it will be the second-order transfer matrix from the last R1 update. The comments about phase space weighting, made in connection with the T1 constraint, are equally valid for the T2 constraint, provided the phase space factors are obtained from the beam matrix at the position of the R2 update.

Second-order σ(BEAM) matrix fitting constraint

Five parameters must be specified for a constraint on the second-order contributions to a beam matrix diagonal element $\sigma_{ii}$.

1 - Type code 10.0

2 - Code digit (i).

3 - Code digit (i).

4 - The number 0.

5 - Desired accuracy of the fit (standard deviation).

If, for example, one wished to minimize the net contributions of second-order aberrations to the horizontal divergence, one would insert the following card:

10. 2. 2. .0 .01 ;

The quantity that is minimized is the net increase due to second-order terms in the second moment of the beam about the origin. This quantity is treated as the chi-squared of the problem, so the only meaningful desired value for the fit is zero. The square root of this quantity is printed in the output. It is computed using the $R_2$ matrix. Therefore, once again, one must not include any element which updates the $R_2$ matrix. Centroid shifts must not be inserted when doing second-order fitting, even immediately following the beam card.

The second-order image of the initial beam centroid at some later point in the beam is not necessarily the beam centroid at the later point. The parameters printed by TRANSPORT are the new centroid position and the beam matrix about the new centroid. One must therefore look at both of these to observe the effects of the fitting procedure. It may even happen that an improvement in one parameter will be accompanied by a slight deterioration in the other.

The beam profile at any point is a function of the initial beam parameters. One may therefore impose weights on the effect of the various aberrations by the choice of parameters on the BEAM card. One might, for example, adjust the strength of the correction of the chromatic aberrations by the choice of the $\Delta p/p$ parameter. In particular, when using a BEAM constraint, one should not attempt to minimize or eliminate chromatic aberrations if $\Delta p/p$ is set equal to zero on the beam card (type code 1.0).

Correlations (the 12.0 card) may also be included in the initial beam specification.

Sextupole strength constraints

Five parameters must be specified for a constraint on sextupole strength.

1 - Type code 10.0
2 - Code digit 18.
3 - Code digit 0.
4 - The number 0.
5 - Desired maximum sextupole field strength.

A single sextupole constraint card applies to all sextupoles which follow. The maximum field strength is treated as a standard deviation and may be exceeded on an optimal fit.

One can employ this constraint to find the optimal locations for sextupoles. By placing inversely coupled drift lengths before and after the sextupole its longitudinal position may be varied. By constraining the field strength the sextupole can be slid to a position where the coupling coefficients to the aberrations will be largest. One will need to experiment with adjusting the maximum field strength to achieve the best configuration.

Internal constraints

A set of upper and lower bounds on the value of each type of parameter is in the memory of the program. If a correction is computed for a parameter which would take its value outside this range, it is reset to the limit of the range. The current limits are:

| Type code | Limits |
|---|---|
| 1.0 | 0 < input beam |
| 2.0 | -60 < pole-face rotation < 60 (deg) |
| 3.0 | 0 < drift |
| 4.0 | 0 < magnet length |
| 5.0 | 0 < quad length |
| 20.0 | -360 < beam rotation < 360 (deg). |

These limits apply only when a parameter is being varied. Fixed values that exceed this range may be used as desired.

These constraints were included to avoid physically meaningless solutions.

Corrections and covariance matrix

When the program is fitting, it makes a series of runs through the beam line. From each run it calculates the chi-squared and the corrections to be made to the varied parameters. For each iteration a single line is printed containing these quantities.

The program calculates the corrections to be made using a matrix inversion procedure. However, because some problems are difficult, it proceeds with caution. The corrections actually made are sometimes a fixed fraction of those calculated. This fraction, used as a scaling factor, is the first item appearing on the line of printed output. The second factor is the chi-squared before the calculated corrections are made. Following are the corrections to be made to the varied parameters. They are in the order in which they appear in the beam line. If several parameters are coupled, they are considered as one and their position is determined by the first to appear.

When convergence has occurred, the final value of the chi-squared and the covariance matrix are printed. The covariance matrix is symmetric, so only a triangular matrix is shown. The diagonal elements give the change in each varied parameter needed to produce a unit increase in the chi-squared. The off-diagonal elements give the correlations between the varied parameters.

The appearance of the chi-squared and covariance matrix is:

*COVARIANCE (FIT $\chi^2$)

$$\begin{matrix} \sqrt{C_{11}} & & & \\ r_{12} & \sqrt{C_{22}} & & \\ \vdots & & \ddots & \\ r_{1n} & \cdots & r_{n,n-1} & \sqrt{C_{nn}} \end{matrix}$$

For more details on the mathematics of the fitting, the user should consult the Appendix. For an example of the output of the program he (or she) should refer to the section on output format.