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DECAY TURTLE*

(Trace Unlimited Rays Through Lumped Elements)

A COMPUTER PROGRAM FOR SIMULATING CHARGED PARTICLE BEAM
TRANSPORT SYSTEMS, INCLUDING DECAY CALCULATIONS

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1. INTRODUCTION

TURTLE is a computer program designed to simulate charged particle beam transport systems. It allows evaluation of the effect of aberrations which exist in beams with small phase-space volume. These include higher-order chromatic aberrations, effects of nonlinearities in magnetic fields, and higher-order geometric aberrations due to the accumulation of second-order effects. The beam at any point in the system may be represented by one- and two-dimensional histograms.

The original version of TURTLE was created by D. C. Carey at Fermilab.* The major part of this program and of the present manual are his work.

In this version of TURTLE, which was originally developed at CERN,** and called DECAY TURTLE, provision has been made to include particle decay, following the parent particles and up to two kinds of daughter particles through the beam line.

* TURTLE (Trace Unlimited Rays Through Lumped Elements), A computer program for simulating charged particle beam transport systems, by D. C. Carey (National Accelerator Laboratory, Batavia, IL 60510, USA, Report NAL-64, December 1971).

** DECAY TURTLE, CERN 74-2, February 1974. K. L. Brown and Ch. Iselin.

2. GENERAL THEORY

2.1. Uses and Limitations of the Matrix Approach

As in TRANSPORT, we represent the position and direction of travel of a particle entering a beam line via a vector with six coordinates^{1),2)}

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{bmatrix} . \quad (1)$$

The coordinates x and y represent, respectively, the horizontal and vertical displacements at the position of the particle, and x' and y' represent the angles with the axis of the beam line in the same planes. The quantity ℓ represents the longitudinal position of the particle relative to a particle travelling on the magnetic axis of the system with the central momentum designed for the system. The remaining quantity $\delta = (p-p_0)/p_0$ gives the fractional deviation of the momentum of the particle from the central design momentum of the system.

The effect of the passage of a particle across a magnetic element or a drift space is represented to first order by a transfer matrix R . The coordinates $X(1)$ of the particle at the end of the element are then given in terms of those at the beginning $X(0)$ as

$$X(1) = RX(0) . \quad (2)$$

The effect of successive elements, each with its own R matrix is given by a total R matrix $R(t)$ which is equal to the product of the individual R matrices.

$$R(t) = R_n R_{n-1} \dots R_2 R_1 \quad . \quad (3)$$

The first columns of such a matrix are obtained by solving a differential equation of the form

$$g'' + k^2 g = 0 \quad (4)$$

with appropriate boundary conditions, and taking either the value or the derivative of the solution. The differentiation is with respect to distance along the beam line and k^2 is a function only of that quantity.

The last two columns are obtained by solving the equation

$$g'' + k^2 g = f \quad , \quad (5)$$

where f is a driving term given in terms of quantities which depend only on the position along the beam line and single factors of solutions to either Eq. (4) or (5). Equation (5) may be solved by a Green's function by writing

$$g(t) = \int_0^t G(t, \tau) f(\tau) d\tau \quad . \quad (6)$$

If we treat Eq. (2) as the first term of a power series expansion of $X(1)$ in terms of $X(0)$ we may write further terms as follows:

$$x_i(t) = \sum_j R_{ij} x_j(0) + \sum_{j,k} T_{ijk} x_j(0) x_k(0) \quad . \quad (7)$$

Here our definition of the T matrix differs from Brown's¹⁾ in that ours is rectangular while Brown's is triangular. The use of a rectangular matrix is more easily extended to the study of higher-order effects. The difference between the two matrices is that the off-diagonal elements of the rectangular matrix are exactly half those of the triangular matrix.

As in the case of the R matrix, the T matrix, for a succession of elements, may be written in terms of the R and T matrices of the individual elements via a product. The T matrix T(t) for a succession of two elements is written in terms of the individual R and T matrices as follows:

$$T(t)_{ijk} = \sum_{\ell} R_{i\ell}^{(2)} T_{\ell jk}^{(1)} + \sum_{\ell m} T_{i\ell m}^{(2)} R_{\ell j}^{(1)} R_{mk}^{(1)} \quad (8)$$

Notice that at no point in the evaluation of a T matrix for a beam line are two individual T matrices multiplied together. Such a multiplication would yield terms of order higher than second. Since in a matrix approach one characterizes the effects of the beam line according to order, and we are here expanding to second order, we must consistently truncate our results to second order.

In a matrix approach the beam itself may also be represented by a matrix σ . The phase space occupied by the beam is taken to be an ellipsoid in six dimensions. Usually this matrix is initially taken to be diagonal with the square roots of the diagonal elements being equal to the semi-axes of the ellipsoid. The beam ellipsoid at a later point is obtained, using the R matrix, as

$$\sigma_1 = R \sigma_0 R^T \quad . \quad (9)$$

The diagonal elements are now the squares of half the maximum extent of the ellipsoid in a given dimension. The off-diagonal elements give the correlation between the coordinates, as for a tilted ellipse. A phase-space envelope which is initially ellipsoidal continues to be so only when we limit our consideration to first-order effects. When higher orders are considered the elements of the beam matrix may be taken as second moments of a distribution, but give us no information as to the actual shape of the phase-space envelope.

The matrix method is indispensable for fitting and is the only reasonable way to obtain an initial design of a beam. It is also unsurpassed as a tool to determine individual second-order influences on particle trajectories and minimizing such aberrations. Its limitations are in the representation of the beam phase space and the fact that the results are limited by the order of the Taylor's series expansion.

2.2. Ray-Tracing Methods

A true ray-tracing program computes the trajectory of a particle through a magnetic field directly. It, therefore, does not distinguish among different order effects on a ray, but represents all orders to the accuracy of the numerical integration of the equations of motion. Such an approach is also useful for determining individual contributions to aberrations. If, however, one wishes to represent the phase space occupied by the beam, it is necessary to run large numbers of rays

through the system to obtain a reasonable population. For a complicated system this approach can prove time consuming. In addition it is often unnecessary, as in the case of beams possessing a small phase-space volume.

2.3. Theory of Ray Tracing Through Lumped Elements

In order to describe the procedure employed in TURTLE, we must explain what is meant by the distinction between local and global classification of aberrations according to order. If we once again let X be a vector giving the coordinates and direction of a particle (henceforth referred to as a ray), and expand the differential equation of motion of a particle passing through a magnetic field in powers of X , we obtain, deleting subscripts,

$$DX + EX^2 + FX^3 + \dots = 0 \quad (10)$$

The coefficients D , E and F are matrices, with the diagonal terms of D being differential operators

$$DX = \left[\frac{d^2}{dt^2} + k^2 \right] X \quad (11)$$

The first-order transfer matrix R is obtained by solving the differential equation to first order, i.e.

$$DX = 0 \quad (12)$$

The second-order transfer matrix T is now obtained from the second-order coefficients in the differential equation via a Green's function

$$T(t,0) = \int_0^t G(t,\tau) E(\tau) X^2(\tau) d\tau \quad , \quad (13)$$

where $X(t)$ is obtained from the first-order transformation

$$X(t) = R(t,0) X(0) \quad . \quad (14)$$

We see that to second order, the coefficients of a given order in the differential equation of motion give rise to transfer matrices of the same order. If, however, we wish to go further and ask for third-order transfer matrices $U_{ijk\ell}$, then we find that

$$\begin{aligned} U &= \int_0^t G(t,\tau) F(\tau) X^3(\tau) d\tau \\ &+ \int_0^t G(t,\tau_1) E(\tau_1) X(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2) E(\tau_2) X^2(\tau_2) d\tau_2 d\tau_1 \\ &+ \int_0^t G(t,\tau_1) E(\tau_1) \int_0^{\tau_1} G(\tau_1,\tau_2) E(\tau_2) X^2(\tau_2) d\tau_2 X(\tau_1) d\tau_1 \quad . \end{aligned}$$

The first term contains the third-order coefficients in the differential equation, while the second and third contain the second-order coefficients. Elements of transfer matrices beyond second order therefore involve not only coefficients of the differential equation of the same order, but also accumulations of lower-order terms. We, accordingly, call a classification by order of aberrations according to their appearance in the equation of motion as a local classification.

A classification of terms by appearance in a transfer matrix will be called global.

A matrix approach as defined in Section 2.1 classifies aberrations by order both locally and globally. Such an approach is necessary for any order if one is interested in correcting aberrations of that order. A true ray-tracing program as explained in Section 2.2 need not classify aberrations by order at all.

In the lumped element approach to ray tracing, we classify aberrations by order locally but not globally. The passage of a ray across an individual-element is given by a transformation which yields the output ray directly from the input ray. A large number of rays can then be passed through a system in a short time. The coordinates of the rays can then be collected at any point in the beam line and histograms can be generated. We can then exhibit the phase space occupied by the beam explicitly, and do not have to depend on an ellipsoid formalism.

The use of the lumped element approach permits the inclusion of many sorts of aberrations, but effectively precludes the use of others. To understand how TURTLE works we must examine the different types of local aberrations. We characterize local aberrations into four types: chromatic effects, geometric effects, magnetic field effects, and mixtures of any of the first three types.

Chromatic effects are due to a deviation of the momentum of a ray from the central momentum of the beam line. An example would be the chromatic aberration of a quadrupole, where the focusing strength depends on the

momentum of the particle. Geometric effects are associated with the phase space accepted by the beam line. A trajectory entering a bending magnet at an angle to the central trajectory traverses a different path length in the field and is therefore bent through a different angle than is the central trajectory. This effect gives rise to second- and higher-order geometric aberrations for bending magnets. Another example occurs in the normal treatment of quadrupoles. The harmonic oscillator approximation for the motion of a charged particle in a quadrupole is based on a small angle approximation. Corrections for this approximation involve geometric aberrations of third and higher order. Magnetic field effects are due to nonlinearities appearing in the expansion of the magnetic field in terms of the transverse coordinates of the beam line. An example is the effect of a sextupole for on-momentum rays. Mixed effects represent combinations of the above effects. One example might be the momentum dependence of the effect of a sextupole.

The intent of creating TURTLE was to examine chromatic aberrations and the effects of nonlinearities in magnetic fields to all orders, and to evaluate the effect of slits and apertures and to represent the beam phase-space distribution, including effects of second and higher orders. Geometric effects are considered locally only to second order, but higher order global effects will appear due to the accumulation of second-order effects. It was not possible to achieve this goal rigorously, so we explain below what was done in each case and why the approximations used should be valid for beams possessing a small monoenergetic phase-space volume.

Transfer matrix elements for quadrupoles and sextupoles are evaluated directly for each ray from the actual momentum of the ray, and are, therefore, exact to all orders in chromatic effects. However, in bending magnets, chromatic effects are evaluated only to second order. But in high-energy separated function beams the net focusing effects of bending magnets are small compared to quadrupole contributions, so it is usually sufficient to include only second-order chromatic corrections for bending magnets.

In decay problems, a broader momentum band of particles is generated and must be tracked through the beam line. In DECAY TURTLE we have therefore modified the treatment of bending magnets to improve the precision of calculation of the chromatic (momentum dependent) terms.

Because of the small phase-space volume occupied by typical high-energy beams, the third and higher order geometric effects of a given element will be small. However, higher-order global terms may occur due to cumulative effects of second-order terms. The large lever arms for aberrations in beams at high energies will enhance the importance of such cumulative effects relative to local higher-order geometric effects.

The inclusion of apertures and slits and the representation of the beam phase space with histograms allow a further step in the realistic representation of a beam.

Below we explain how to use the program and further describe each element available.

3. USE OF TURTLE

TURTLE is designed to be run using the same deck of data cards as was used for TRANSPORT.³⁾ Typically one will start with a deck of cards containing approximate values of the final parameters to use in obtaining a fit to the desired constraints. Once the final set of parameters is known, one alters this deck to include them for purposes of studying second-order aberrations and misalignments. With a few changes this deck may be used as input for TURTLE. The data format is free field and the deck structure is the same. Below we discuss the deck structure and each of the type codes. For completeness we include many things that are unchanged from TRANSPORT.

3.1. Structure of the Deck

(1) Title card

The first card of the deck contains the title of the run enclosed in single quotes. No other item should appear on this card.

(2) Number of rays

The second card in a TRANSPORT deck contains an integer which serves as an indicator. When using TURTLE this card contains an integer indicating how many rays one wishes to run through the system. Naturally one will want to choose this number sufficiently high to obtain good statistics. A number of rays equal to several thousand should be quite reasonable for any beam. The only limitation on the number of rays that can be run will be the use of computer time. The user will eventually wish to select this number on the basis of his own experience.

(3) Elements

The elements with their type codes and appropriate parameters and labels are entered in sequence just as in TRANSPORT. Each element must be followed by a semicolon. The labels are not used by TURTLE but may be retained and will appear in the output. This will enable the user to compare his output from TURTLE with that from TRANSPORT. There are additional type codes indicating the creation of histograms. All type codes, including those for histograms, will be explained below.

(4) Comments

Comments may be placed in the data deck before any type code entry. They are indicated by enclosing in parentheses.

(5) Sentinel

The input data is terminated with a SENTINEL card.* The program, after reading and initially processing the data, runs the specified number of rays through the beam line and collects and plots histograms.

As in TRANSPORT, several problem decks may be stacked together and run in the same job. However, the problems cannot depend on each other, as is possible in a TRANSPORT run via the use of the ± 1 indicator card. Instead each problem deck must specify a complete beam line.

* Note that in some versions of TURTLE, a double SENTINEL is required at the end of the deck before execution will begin.

3.2. Type Codes

BEAM: Type Code 1.0

Type code 1.0 can be used either to represent the initial phase space of the beam or a random addition to the beam. There are either eight or nine parameters on the beam card, as follows:

- 1 - Type code 1.0.
- 2 - One half the horizontal extent x of the beam
(cm in standard units).
- 3 - One half the horizontal angular divergence x'
(mr in standard units).
- 4 - One half the vertical extent y of the beam
(cm in standard units).
- 5 - One half the vertical angular divergence y'
(mr in standard units).
- 6 - One half the longitudinal extent of the beam.
In DECAY TURTLE this coordinate is not affected
as the ray proceeds down the beam line.
- 7 - One half the momentum spread δ of the beam
(in units of percent)

$$\delta = \frac{p - p_0}{p_0} = \frac{\Delta p}{p_0}$$

- 8 - The central design momentum of the beam
(GeV/c in standard units).
- 9 - The code digit id indicating a random addition
to the beam. If one is specifying the initial
phase space this entry must be absent.

In TRANSPORT and original TURTLE, the code digit id for a random addition to the beam is always zero. However in DECAY TURTLE, id must identify the particle to which the random addition is to be made as follows:

id = 0 adds a random addition to the parent particles only.

id = 100 adds a random addition to the charged daughters only.

id = 200 adds a random addition to the neutral daughters only.

Each ray is chosen at random with its coordinates constrained to lie within the limits specified on the beam card. In addition, the x and y coordinates are constrained to lie within an upright ellipse whose semi-axes are the quantities given on the beam card. The angular coordinates x' and y' are also constrained to lie within a similar ellipse. This prevents the implicit representation of a preferred direction in space by the choice of coordinates. In other words, targets may be circular and scattering is isotropic. Further restrictions on the phase space, such as the use of a multi-dimensional ellipsoid were felt not to be physically realistic. This is illustrated by the fact that the target is the same size for particles of different momentum.

If we let x_0 , x_0' , y_0 , y_0' , l_0 and δ_0 represent the maximum extents of the beam envelope in each coordinate, then the additional restriction normally imposed by TURTLE is equivalent to the two equations

$$\left(\frac{x}{x_0} \right)^2 + \left(\frac{y}{y_0} \right)^2 \leq 1 ,$$

$$\left(\frac{x'}{x_0'} \right)^2 + \left(\frac{y'}{y_0'} \right)^2 \leq 1 .$$

The hypervolume of this four-dimensional region is equal to $\pi^2 x_0 x_0' y_0 y_0'$.

By contrast the hypervolume of the region specified by the single ellipsoidal condition

$$\left(\frac{x}{x_0} \right)^2 + \left(\frac{x'}{x_0'} \right)^2 + \left(\frac{y}{y_0} \right)^2 + \left(\frac{y'}{y_0'} \right)^2 \leq 1$$

is equal to $(\pi^2/2)x_0 x_0' y_0 y_0'$, or exactly one half that of the region specified by the first set of conditions. This additional volume occurs near the boundaries of the region specified in the two-ellipse condition. When the multi-dimensional distribution is projected onto one or two dimensions the ellipsoidal condition yields a distribution which is very sparse near the edges. The two-ellipse condition yields a projection where the edges of the distribution are much more sharply defined and is thus much better suited to beam line studies.

Note that in the SLAC version of TURTLE, a Gaussian distribution for the phase space parameters x_0 , x_0' , y_0 , y_0' , and δ_0 is specified by placing a minus sign in front of x_0 in the type code 1.0 card, i.e., the type code 1.0 card should appear as follows:

1.0 - x_0 x_0' y_0 y_0' δ_0 P_0 ;

The entries for x_0 , x_0' , y_0 , y_0' and δ_0 are one sigma of the Gaussian distribution desired.

For a random addition to the beam, the coordinates are chosen in the same manner as described above, but they are then added to the coordinates of the ray at the point where the random addition is made. Because of the random selection of both the original ray and the addition to it the effective widths of the distributions in any coordinate will add in an r.m.s. fashion. It should be remembered that the parameters on the card indicating the random addition specify the maximum amplitude of the scattering. If the element immediately preceding a random addition is a slit (type code 6.0) with the same particle id the random addition will be made only if the ray hits the slit. If the ray passes through the slit its coordinates will be unchanged and it will continue unperturbed. Note that for original TURTLE the particle id is zero and is the same by definition on both type code entries.

FRINGING FIELDS AND POLE-FACE ROTATIONS FOR BENDING MAGNETS:

Type Code 2.0

Type code 2.0 specifies the pole-face rotation of a bending magnet and causes the effect of the fringing fields to be calculated. It should immediately precede or follow a type code 4.0 representing a bending magnet. There are two parameters:

- 1 - Type code 2.0.
- 2 - The pole-face rotation angle (in degrees).

Even if the pole-face rotation angle is zero, it is necessary to insert a 2.0 card to take the fringing field into account. When the program is run the effects of the fringing field and the bending magnet are included in a single transfer matrix for the entire magnet. It is therefore not possible to insert a histogram between a type code 2.0 and 4.0 element. More information about the transfer matrix will be found in the description of the 4.0 type code.

The matrix elements for pole-face rotations and fringing fields of bending magnets are calculated to second order only for both versions of TURTLE. See the section under type code 4.0 for a description of the method of calculation for the magnetic fields for the interior portion of sector bending magnets.

If the bending magnets to be specified are rectangular as seen from the top, then the sum of the input and output pole-face rotation angles will equal the bend angle. Several options exist for automatically setting the pole-face rotation angles in terms of the bend angle. Such options are given by entries of 13. 40. ; through 13. 43. ; inclusive.

Even when one of these options is selected, the 2. element must be included in the data before and after type code 4. entry if fringing field effects are to be calculated. Details of the various options are found in the section describing type code 13.

DRIFT SPACE: Type Code 3.0

A drift space is a region containing no magnetic elements. Two parameters are required:

- 1 - Type code 3.0.
- 2 - Length (normal unit is meters).

SECTOR BENDING MAGNET: Type Code 4.0

For a sector bending magnet the entrance and exit faces of the magnet are straight and perpendicular to the central axis of the beam. Other types of magnets may be represented by using a type code 2.0 and any of the bending magnet parameters included under type code 16.0. A type code 4.0 requires four parameters:

- 1 - Type code 4.0.
- 2 - The effective length of the central trajectory through the magnetic field (normal unit is meters).
- 3 - The field strength B_0 along the central trajectory (normal unit is kilogauss).
- 4 - The field gradient n in dimensionless units.

$$n = - \frac{\rho_0}{B_0} \frac{\partial B}{\partial x} (x, 0, t)$$

where $\rho_0 = p_0/(qB_0)$, p_0 is the central momentum at the beam, q the charge of the particle, and ρ_0 the radius of curvature of the central trajectory.

If a 13. 48. ; entry is inserted in the data, all subsequent bend magnet entries are taken to be expressed in terms of L , α , and n . The four parameters to be specified are then:

- 1 - Type code 4.0 (specifying a wedge bending magnet).
- 2 - The (effective) length L of the central trajectory in meters.
- 3 - The bend angle α in degrees.
- 4 - The field gradient (n -value).

The units for bend angle may be changed via a 15. 7. element preceding the beam card.

At a later point one can revert to the normal description in terms of L, B, and n by the insertion of a 13. 47. ; element.

If second order (see type code 17.0) is not specified, a ray with the beam central momentum is transformed through the magnet using the first-order transfer matrix R so that $X(1) = RX(0)$. If the ray does not have the central momentum we use an off-momentum R matrix which is obtained from the ordinary first-order matrix R^0 and the chromatic elements of the second-order matrix T via the equation

$$R_{ij} = R_{ij}^0 + T_{ij6} \frac{\Delta p}{p_0}$$

where $\Delta p/p_0 = (p-p_0)/p_0$ is the fractional deviation from the central momentum. If a type code 17.0 card is included in the deck, second-order geometric effects will also be included in the calculations. These correspond to the matrix elements T_{ijk} , where j and k are both equal to or less than four.

In DECAY TURTLE, a complete second-order calculation is automatically invoked when a type code 60.0 card is inserted calling for decay calculations to be made.

In DECAY TURTLE, the treatment of momentum-dependent terms in sector bending magnets differs from that in original TURTLE as follows. Consider the equations of motion as given on Page 34 of Ref. 1:

$$x'' - h(1 + hx) - x'(hx' + h'x)$$

$$= \frac{p_0}{p} T' \left\{ (1+hx) [-h + nh^2x - \beta h^3x^2 + \frac{1}{2}(h''-nh^3+2\beta h^3)y^2] + h'yy' + \dots \right\}$$

$$y'' - y'(hx' + h'x)$$

$$= \frac{p_0}{p} T' \left\{ -h'x'y' - (1 + hx)[nh^2y - 2\beta h^3xy] + \dots \right\}$$

We now introduce the second-order expansion

$$T' = 1 + hx + \frac{1}{2}(x'^2 + y'^2) + \dots$$

and redefine the fractional momentum charge to be

$$\epsilon = \frac{p - p_0}{p} = \frac{\delta}{1+\delta} \quad \text{where} \quad \delta = \left(\frac{p - p_0}{p_0} \right) \quad \text{from Ref. 1 ;}$$

then the equations of motion may be rewritten to second order as follows:

$$x'' + h^2(1-n)x = h\epsilon + h^3(2n-\beta-1)x^2 + h'xx + \frac{1}{2}hx'^2 + h^2(2-n)x\epsilon \\ + \frac{1}{2}(h''-nh^3+2\beta h^3)y^2 + h'yy' - \frac{1}{2}hy'^2 + \dots$$

$$y'' + h^2ny = 2h^3(\beta-n)xy + h'xy' - h'x'y + hx'y' + nh^2y\epsilon + \dots$$

Note that these equations have the same general form as those given in Ref. 1, except that here no ϵ^2 term appears in the x equations. Using the same method as in Ref. 1, we have solved these equations and find that all matrix elements remain the same, except for T_{166} and T_{266} which now have the form:

$$T_{166} = h^2(2-n)I_{16} + h^3(2n-\beta-1)I_{166} + \frac{1}{2}h^3I_{122}$$

$$T_{266} = h^2(2-n)I_{26} + h^3(2n-\beta-1)I_{266} + \frac{1}{2}h^3I_{222} - hd_x dx'$$

Decay TURTLE uses these modified equations, except for uniform field magnets ($n=0, \beta=0$) in which case the following exact formula set is used to describe the trajectories:

$$x_2 + \rho_0 = (x_1 + \rho_0)\cos\alpha + \rho[\cos\theta_2 - \cos(\theta_1 + \alpha)]$$

$$\rho \sin\theta_2 = \rho \sin(\theta_1 + \alpha) - (x_1 + \rho_0) \sin\alpha$$

$$y_2 = y_1 + y_1' \rho(\alpha + \theta_1 - \theta_2) \cos\theta_1$$

$$y_2' = y_1' \frac{\cos\theta_1}{\cos\theta_2}$$

where:

ρ_0 is the radius of curvature of the central trajectory having a momentum p_0 .

ρ is the radius of curvature of an arbitrary trajectory having a momentum p .

α is the total bending angle of the central trajectory.

θ_1 is the entrance angle made between the central trajectory and the projection of an arbitrary trajectory onto the bend (mid-) plane of the magnet.

θ_2 is the exit angle made between the central trajectory and the projection of an arbitrary trajectory onto the bend (mid-) plane of the magnet.

y_1' is the tangent of the angle ϕ_1 made between the central trajectory and the projection of an arbitrary trajectory onto a plane perpendicular to the midplane, at the entrance of the magnet (see Fig. 1).

y_2' is the tangent of the angle ϕ_2 made between the central trajectory and the projection of an arbitrary trajectory onto a plane perpendicular to the midplane, at the exit of the magnet (see Fig. 1).

x_1, y_1, x_2, y_2 are the x and y coordinates of the arbitrary trajectory at the entrance and exit planes of the magnet relative to the central trajectory.

This set of exact equations is valid for uniform field sector (wedge) bending magnets (ignoring entrance and exit fringing fields) and can be verified using the constructions in Figs. 1 and 2.

No fringing-field effects or pole-face rotations are included in type code 4.0 entries. To include them type code 2.0 cards must be inserted before and after such a magnet, even if the pole-face rotation angle is zero.

Original TURTLE assembles a single transfer matrix for each bending magnet including the fringing fields. It then transforms each ray using this assembled transformation matrix. In DECAY TURTLE, the tracking through a bending magnet normally consists of three steps:

- tracking through the entrance fringing field, if present,
- tracking through the idealized sector bending magnet, and
- tracking through the exit fringing field, if present.

This three-step process has been used because, if a decay occurs within the magnet, the transform must be split correctly at the point of decay.

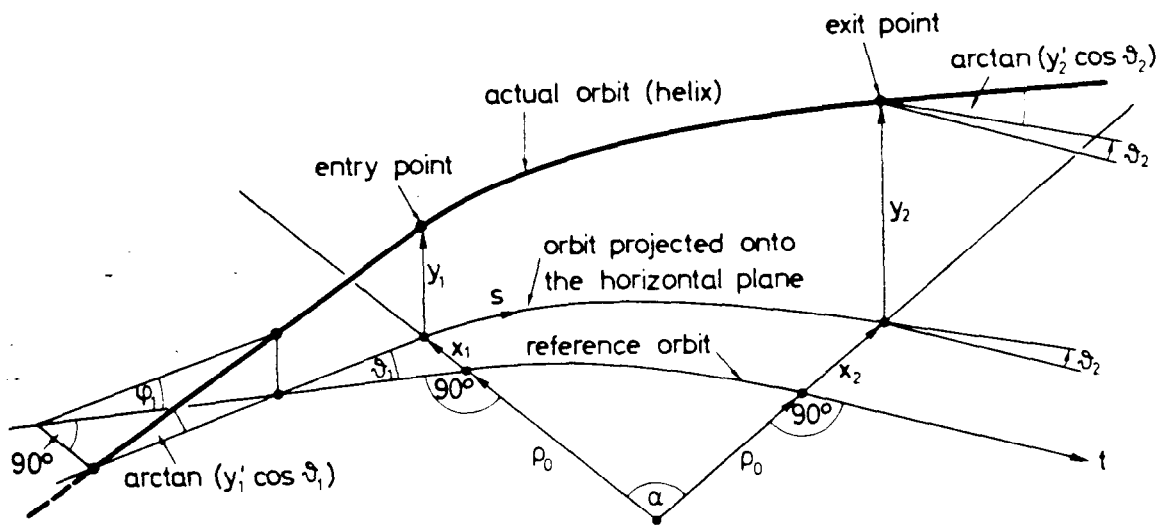


Fig. 1. View of the three-dimensional orbit in a homogeneous field sector bending magnet.

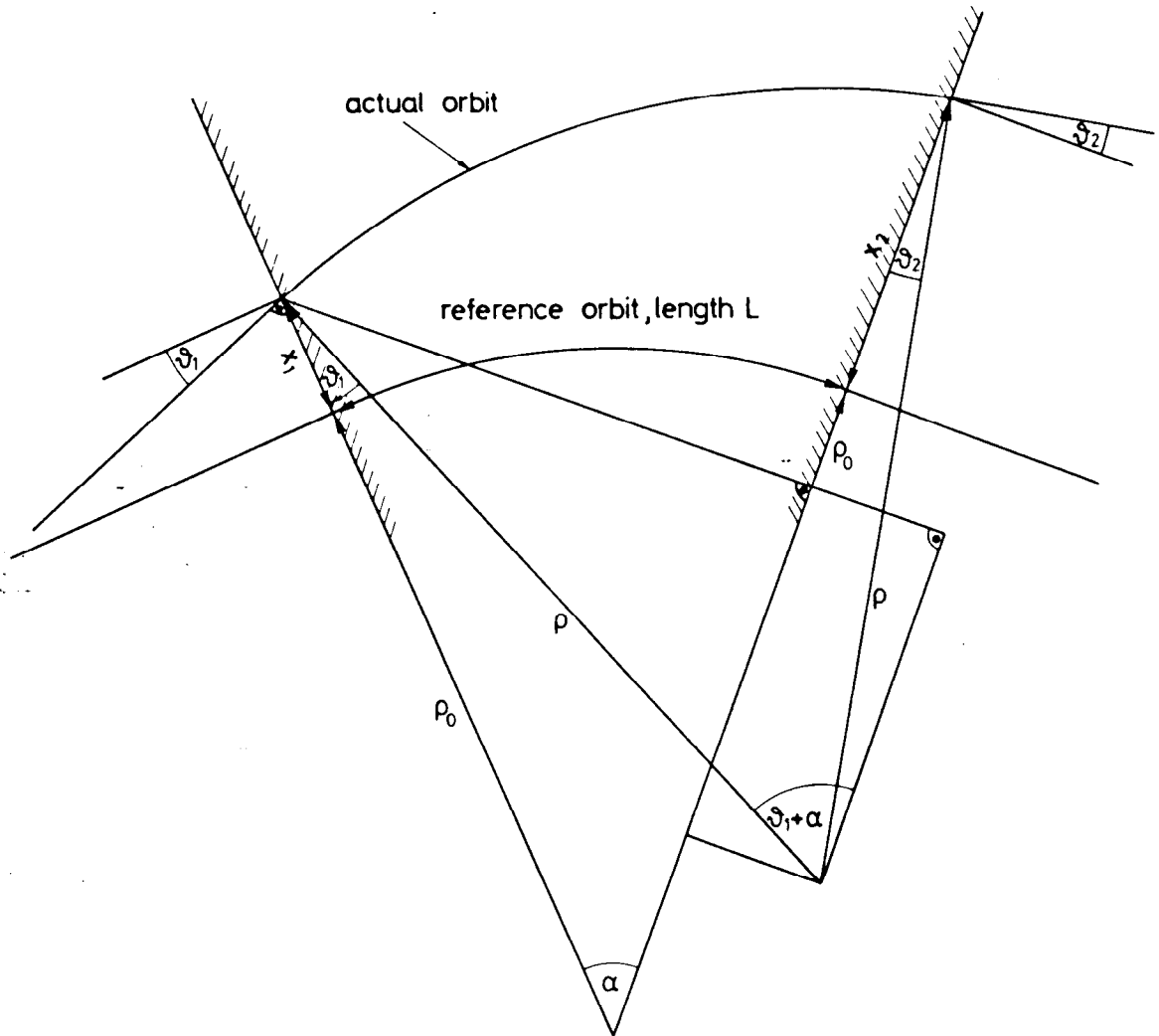


Fig. 2. Projection of Fig. 1 onto the magnetic midplane.

QUADRUPOLE: Type Code 5.0

A quadrupole requires four parameters for its specification:

- 1 - Type code 5.0.
- 2 - The effective length of the quadrupole field
(normal unit is meters).
- 3 - The pole tip field, B_0 , a positive field
indicating a horizontally-focusing quadrupole
(normal unit is kilogauss).
- 4 - Pole tip half aperture a (normal unit is cm)
The gradient is obtained by dividing the pole
tip field by this aperture.

For a perfect quadrupole the first four components of the ray vector $X = (x, x', y, y', \delta)$ are carried through the quadrupole via the transformation matrix

$$\begin{pmatrix} \cos k\ell & \frac{1}{k} \sin k\ell & 0 & 0 \\ -k \sin k\ell & \cos k\ell & 0 & 0 \\ 0 & 0 & \cosh k\ell & \frac{1}{k} \sinh k\ell \\ 0 & 0 & k \sinh k\ell & \cosh k\ell \end{pmatrix}$$

where $k = qB_0/ap$, p is the actual momentum of the ray, and q is the charge of the particle. This matrix is for a horizontally-focusing quadrupole. A matrix for a vertically-focusing quadrupole is obtained by exchanging the two submatrices occurring on the diagonal. Since this matrix is evaluated for each ray using the actual momentum of the ray it is valid to all orders in chromatic effects.

Higher-order multipole effects for a realistic quadrupole may also be introduced via a 16.0 type code entry. Multipoles up to and including a 40-pole may be included. In original TURTLE the multipole aberrations are taken to be lumped at the longitudinal midpoint of the quadrupole. The ray is transformed half-way through the quadrupole, perturbed by the multipole aberrations, then transformed through the remainder of the quadrupole. If no multipole aberrations are included the ray passes through the quadrupole in a single step. The multipole component strengths are normalized to unit gradient and need be entered only once for similar but differently excited quadrupoles. The effect of the multipoles is also calculated from the actual momentum of the ray, making this effect correct to all orders chromatically.

While the quadrupole matrix elements are computed in the same way in both versions of TURTLE, DECAY TURTLE applies the multipole aberrations half at the entrance and half at the exit of the quadrupole rather than at its center. However, this does not change the coding procedure for the user.

SLIT: Type Code 6.0

A slit entry requires three parameters:

- 1 - Type code 6.0.
- 2 - The code number $i + id$ for the coordinate and the particle identification (see below).
- 3 - The half opening of the slit (in the units for the appropriate coordinate).

In DECAY TURTLE, an elliptic slit is available using five parameters:

- 1 - Type code 6.0.
- 2 - The code number $i + id$ for the first coordinate and the particle identification (see below).
- 3 - The first half axis of the ellipse.
- 4 - The code number j for the second coordinate.
(Any particle identification on this entry is ignored.)
- 5 - The second half axis of the ellipse.

The code digit i or j specifies the coordinate as follows:

- 1 - x coordinate.
- 2 - x' coordinate.
- 3 - y coordinate.
- 4 - y' coordinate.
- 6 - $\delta = (p - p_0)/p_0$.

In original TURTLE the particle identification code id is always zero.

In DECAY TURTLE meaningful values are:

- 0 - parent particles
- 100 - charged daughter particles
- 200 - neutral daughter particles.

If a ray encounters a slit and the ray coordinates lie outside the specified aperture, the ray will be stopped. If a random addition to the beam with the same particle id immediately follows the slit, the particle will not be stopped, but its coordinates will be changed by an amount within the range specified for the random addition. If the ray passes within the specified aperture of the slit, it will be left undisturbed. Note that since in original TURTLE the particle id is always zero on both entries, the random addition will always be associated with the slit as described above.

Example No. 1. A non-symmetric slit, aperture $-60 \leq x \leq 100$

7.	20.	0.	0.	0.	0.	0.	;
6.	1.	80.					;
7.	-20.	0.	0.	0.	0.	0.	;

The two coordinate shifts will cause all particles to be temporarily shifted towards positive x's such that the beam centroid passes through the center of the slit of half aperture 80.

Table 1: Summary of DECAY TURTLE type codes

PHYSICAL ELEMENT	TYPE CODE	2nd ENTRY	3rd ENTRY	4th ENTRY	5th ENTRY	6th ENTRY	7th ENTRY	8th ENTRY	9th ENTRY	
BEAM	1.0	x (cm)	x' (mr)	y (cm)	y' (mr)	z (cm)	delta (percent)	beta	id	
r.m.s. ADDITION TO BEAM ENVELOPE	1.0	delta (cm)	delta' (mr)	delta y (cm)	delta y' (mr)	delta z (cm)	delta delta (percent)	delta beta (GeV/c)		
POLE-FACE ROTATION (for bending magnets)	2.0	ANGLE OF ROTATION (degrees)								
DRIFT	3.0	LENGTH (metres)								
BENDING MAGNET	4.0	LENGTH (metres)	FIELD (kG)	FIELD GRADIENT (G-value)						
BENDING MAGNET (specified by bend angle)	4.0	LENGTH (metres)	ANGLE (degrees)	FIELD GRADIENT (G-value)						
QUADRUPOLE	5.0	LENGTH (metres)	FIELD (kG)	HALF-APERTURE (cm)						
SPLIT	6.0	id + 1	HALF WIDTH							
ELLIPTIC SPLIT	6.0	id + 1	HALF AXIS (cm)	2	HALF AXIS (cm)					
BEAM CENTRUM SHIFT	7.0	SHIFT (x)(cm)	SHIFT (y)(cm)	SHIFT (z)(cm)	SHIFT (x)(cm)	SHIFT (y)(cm)	SHIFT (z)(cm)	SHIFT (delta percent)		
REPEAT SECTION	9.0	NUMBER OF REPEATS (The end of repeat section is signified by a (9, 0, 0) entry)								
APERTURE CONSTRAINT	13.0	id + 10								
POLE FACE ROTATION ANGLE SPECIFICATION	13.0	40 - 43								
BEND MAGNET INPUT SPECIFICATION	13.0	47 - 48								
REFERENCE CHARGE	13.0	108 - 109								
ARBITRARY R MATRIX	14.0	R(I,J)	R(I,J)	R(I,J)	R(I,J)	R(I,J)	R(I,J)	J		
UNITS CONTROL (transport dimensions)	15.0	CODE	LABEL	SCALE FACTOR (if required)						
QUADRATIC TERM ON BENDING FIELD	16.0	3	$(1) + \frac{1}{2} \frac{1}{(L)^2}$	L in units of transverse length (cm)						
HALF-APERTURE OF BENDING MAG. IN x-PLANE	20.0	4	g/2 (cm)							
HALF-APERTURE OF BENDING MAG. IN y-PLANE (gpp)	20.0	5	g/2 (cm)							
FRINGING FIELD CORRECTION COEFFICIENT	20.0	7	K ₁ (dimensionless)							
FRINGING FIELD CORRECTION COEFFICIENT	20.0	8	K ₂ (dimensionless)							
POLE DECAU LENGTH	20.0	9	L _{max} (metres)							
MINIMUM DECAU LENGTH	20.0	109	L _{min} (metres)							
DISION SEPARATION OF OPTIC AXES	20.0	11	L ₁ (metres)							
EVALUATION OF ENTRANCE END OF BENDING MAGNET	20.0	12	1/K ₁ (1/metres)	NEW introduced, a 20.0 type code entry applies to all appropriate (bending) elements in a beam line where meant to be of a new value.						
EVALUATION OF EXIT END OF BENDING MAGNET	20.0	13	1/K ₂ (1/metres)							
CIRCULAR QUAD. APERTURES	20.0	100	DIAGONAL HALF-APERTURE (cm)							
HYPERBOLIC QUAD. APERTURES	20.0	101	DIAGONAL HALF-APERTURE (cm)							
ELLIPTIC QUAD. APERTURES	20.0	102	VERTICAL HALF WIDTH (cm)							
POSITIVE PARTICLE CHARGE	20.0	200	CHARGE (+1 or 0)	"NAME"						
"POSITIVE" DAUGHTER CHARGE	20.0	201	CHARGE	"NAME"						
"NEUTRAL" DAUGHTER CHARGE	20.0	202	CHARGE	"NAME"						
MULTIPOLE COMPONENTS IN QUAD.	20.0	21	NUMBER OF TERMS	AMPLITUDE (normalized to quadrupole)	PHASE ANGLE (degrees)					
SEXTUPOLE CALCULATIONS	20.0	22	LENGTH (metres)	FIELD (kG)	HALF-APERTURE (cm)					
SOLENOID	20.0	23	LENGTH (metres)	FIELD (kG)						
BEAM ROTATION	20.0	24	ANGLE OF ROTATION (degrees)							
DEFINED SECTION	24.0	CODE DIGIT	"NAME"							
ONE-DIMENSIONAL HISTOGRAM	50.0	id + 1	LOWER LIMIT	UPPER LIMIT	BIN SIZE					
TWO-DIMENSIONAL HISTOGRAM (x-COORDINATE)	51.0	id + 1	LOWER LIMIT	UPPER LIMIT	BIN SIZE					
TWO-DIMENSIONAL HISTOGRAM (y-COORDINATE)	52.0	id + 1	LOWER LIMIT	UPPER LIMIT	BIN SIZE					
HISTOGRAM FLAG	53.0	id + 1	HALF WIDTH							
HISTOGRAM VALUE STORAGE	54.0	id + 1	REGISTER NUMBER							
DECAU	60.0	MASS OF PARENTS (GeV)	MASS OF CHARGED DAUGHTERS (GeV)	MASS OF NEUTRAL DAUGHTERS (GeV)	PARENT LIFETIME (nanoseconds)	NEW introduced, a 60.0 type code entry applies to all appropriate (bending) elements in a beam line where meant to be of a new value.				

All type code entries must be terminated by a semicolon (;) to indicate their end, e.g. 11, 20, ;

id stands for particle identification, where
 id = 0 signifies parents
 id = 100 signifies charged daughters
 id = 200 signifies neutral daughters

r.m.s. ADDITION TO BEAM
 An r.m.s. addition to beam
 11 dx dx' dy dy' dz dz' dp dp' id ;

must specify the particles to be affected via
 id = 0 parents
 id = 100 charged daughters
 id = 200 neutral daughters

It will affect all particles of the specified kind unless it is unambiguously provided a slit with the same particle id, in which case it will affect only the particles outside the specified slit aperture.

x, y stand for coordinates, as follows:

i,j	Coordinate	Standard units
1	x	cm
2	x'	mr
3	y	cm
4	y'	mr
5	$\frac{p_x - p_{x0}}{p_0}$	1
6	$\frac{p_y - p_{y0}}{p_0}$	1
8	z (longitudinal length)	m
11	p (particle momentum)	GeV/c
12	cos(theta) (center-of-mass decay angle)	degrees
14	z (decay point)	metres
21	$r = \sqrt{x^2 + y^2}$ (polar radius)	cm
22	$\phi = \arctan(y/x)$ (azimuthal angle)	mr
23	alpha (polar angle)	degrees
24	beta (azimuthal angle)	degrees

Example No. 2. A slit with a window

- 6. 1. 50. ; aperture for parents, no scattering.
- 6. 101. 50. ; aperture for charged daughters with scattering on slit.
- 1. ... 100. ;
- 1. ... 0. ; causes all remaining parents to scatter in the window.

In this above example the neutral daughters are not affected by any of the slit or r.m.s. type code entries.

Example No. 3. A typical neutrino decay channel

- . Decay channel.
- 6. 1. 0. ; Black beam stopper for all parent particles.
- 1. ... 100. ; All muons are scattered in the beam stopper.
- . muon channel.
- 6. 101. 0. ; Black beam stopper for all muons.
- 3. ... ; neutrino channel.
- 6. 201. 200. 3. 150. ; neutrinos are counted only if they hit an elliptic detector having semi-major axes of 200 cm by 150 cm in x and y directions respectively.

SHIFT IN THE BEAM: Type Code 7.0

The phase-space coordinates of all rays in a TURTLE calculation may be shifted by a constant amount (independent of the ray) at a given point in the beam. Seven parameters are needed:

1 - Type code 7.0.

2 to 7 - The amount of the shift in each coordinate.

Units are the units used for that coordinate.

Coordinate No. 5 is not used but is included

for compatibility with TRANSPORT.

As opposed to TRANSPORT, any volume of phase space is permitted with the use of this element. Note that the misalignment (type code 8.0) is not available in TURTLE. But a known misalignment may be simulated by sandwiching an element between two 7.0 cards depicting the misalignment encountered.

APERTURE CONSTRAINTS AND INPUT OUTPUT OPTIONS: Type Code 13.0

To cause the program to observe magnet apertures two parameters are required:

- 1 - Type code 13.0.
- 2 - The code number $id + 10$, where id identifies the kind of particle.

The code value id is always zero for original TURTLE. In DECAY TURTLE, meaningful values for id are:

- 0 - apertures observed for parent particles
- 100 - apertures observed for charged daughter particles
- 200 - apertures observed for neutral daughter particles.

If the 13. $id + 10$. ; card is not present for a given kind of particle, the only apertures affecting these particles are slits. If it is present, the program will terminate all those rays which pass outside the specified apertures of quadrupoles or bending magnets. The quadrupole half aperture is normally taken to be circular and equal to the fourth entry on the type code 5.0 card. One can specify a different aperture for quadrupoles, and make it either circular, elliptical, or hyperbolic through the use of 16. 100. through 16. 102. elements. The apertures for bending magnets are given by the 16. 4. and 16. 5. elements. All aperture constraints are applied both at the beginning and the end of the element to which they apply. See the section describing the use of the 16.0 element for further details on aperture specification.

Pole-face rotation angle specification 40., 41., 42., 43.

The pole-face rotation angle normally specified with type code 2. may alternatively be calculated from the bend angle of the associated bend magnet. The pole-face rotation element must still be present. The options described here merely cause the value of β (the pole-face rotation angle) to be filled in automatically by the program.

The element specifying the means of determination of the pole-face rotation angle must precede the bending magnet specification, including pole-face rotation angle elements, to which it applies. It will remain in effect until the option is respecified via another 13. type code element. Some caution must be taken in the use of these options. If a bend magnet is segmented, then the pole-face rotation angle will be calculated from the bend angle of the adjacent segment.

(13. 41. ;) Both input and output pole-face rotation angles are equal to half the bend angle.

(13. 42. ;) The entry pole-face rotation angle will be zero. The exit pole-face rotation angle will equal the bend angle.

(13. 43. ;) The entry pole-face rotation angle will equal the bend angle. The exit pole-face rotation angle will be zero.

(13. 40. ;) The normal option is restored. The pole-face rotation angles will be read from the data.

Bend magnet input specifications 47., 48.

(13. 48. ;) Bending magnets encountered subsequent to the insertion of this element are to be specified by the length, bend angle, and n value. Thus a bend magnet element will take the form

4. L α n ;

(13. 47. ;) The normal option for specification of a bending magnet is restored by this element. A bend magnet element now takes the form

4. L B n ;

Reference charge 108., 109.

The reference trajectory is normally taken to be a possible path of a particle with unit positive charge and the central design momentum of the beam line. If the charge of the parent particle is respecified to be neutral, then this may no longer be the case. Sweeping magnets may be used to separate neutral parents from directly produced charged particles. The reference trajectory through the sweeping magnets will be that of a neutral particle.

The charge of the reference trajectory is neutral following a 103. 109. ; element. It is restored to unit positive charge by a 13. 108. ; element.

All type code 13.0 entries normally used in TRANSPORT and not mentioned above will be ignored by TURTLE.

ARBITRARY TRANSFORMATION MATRIX: Type Code 14.0

An arbitrary first-order 6×6 matrix transformation may be introduced. There are eight parameters:

- 1 - Type code 14.0.
- 2 to 7 - The elements of the given row of the transfer matrix. The units of these numbers must be chosen to be consistent with the units for the coordinates.
- 8 - The number of the row.

A matrix must be read in one row at a time. An uninterrupted sequence of 14.0 type cards is taken as providing entries to the same matrix. Any unspecified row is taken to be that of the identity matrix. If two successive sets of type code 14.0 cards are to provide elements of successive matrices, they must be separated by a do-nothing entry. An example of such an element is a (3. 0. ;), a drift space of zero length.

Second-order terms may be introduced by including the 22 additional numbers:

- 9 - Continuation code 0.
- 10 to 30 - The 21 second-order matrix elements
T(i11) T(i12) T(i13) T(i14) T(i15) T(i16) T(i22)
T(i23) T(i24) T(i25) T(I26) T(i33) T(i34) T(i35)
T(i36) T(i44) T(i45) T(i46) T(i55) T(i56) T(i66)
in the order given. The letter "i" indicates the row number, and specifies the coordinate to which these matrix elements contribute.

Each set of second-order coefficients accompanies the row of the first-order matrix which contributes to the same coordinate. Once again successive 14.0 elements are taken as giving entries into the same R and T matrices. Any unspecified row is taken as being the same as the identity transformation. This means that the R matrix elements are given by the Kronecker delta and the T matrix elements are all zero.

In decay TURTLE, the arbitrary transform matrix (type code 14.0) should not be used in a region of the beam line where the decay process takes place or an incorrect yield for decay particles will result.

UNITS CHANGES: Type Code 15.0

Units may be changed to any desired. A specification of units change should appear before any other type code in the deck. All subsequent input data should then be consistent with the units used. There are four entries on a 15.0 card:

- 1 - Type code 15.0.
- 2 - A code digit indicating which unit is to be changed.
- 3 - The name of the new unit. This should be enclosed in single quotes and can be at most four characters long.
- 4 - The size of the unit being introduced in terms of the normal unit otherwise used by the program.

Below is a table of the code digits for the units, the quantity to which they apply, and the standard unit used. The various units that may be changed are:

Code Digit	Quantity	Standard TRANSPORT Unit	Symbols Used in Text
1.0	Horizontal and vertical transverse dimensions, and magnet apertures.	cm	x,y
2.0	Horizontal and vertical angles	mr	x',y'
6.0	Momentum spread	percent (PC)	δ
7.0	Bend and pole-face rotation angles	degrees (DEG)	
8.0	Length (longitudinal) of elements and bending magnet pole face curvatures	meters (M)	λ
9.0	Magnetic fields	kG	B
11.0	Momentum and masses of particles used in decay process	GeV/c GeV for masses	p(0)

In certain cases the conversion factor may be omitted and the program will make the required units change by recognizing the symbols used to represent the unit. The automatic units changes available are the same as those in TRANSPORT and the reader is referred to the TRANSPORT manual for a complete listing of them.

MULTIPOLE ABERRATIONS IN QUADRUPOLES AND OTHER SPECIAL PARAMETERS:

Type Code 16.0

A number of parameters other than those described so far may be used to give further information about quadrupoles and bending magnets. A parameter introduced on a 16.0 card applies to all succeeding elements. It may be changed or reset to zero by introducing another 16.0 type card. Four parameters may be supplied.

- 1 - Type code 16.0.
- 2 - Code digit indicating nature of parameter(s).
- 3 - First special parameter.
- 4 - - Second special parameter (if needed).

Below we describe each of the special parameters available. Some of them are the same as those available in the TRANSPORT program and we refer the reader to that manual for further discussion.³⁾

Coding for multipole aberrations

-N. A negative code digit indicates a multipole aberration in a quadrupole. Code digit -N indicates a 2N-pole. Multipoles up to and including a 40-pole may be included. The first special parameter is equal to B_N/ga^{N-1} , where B_N is the pole-tip field due to the 2N-pole, a is the half aperture, and g is the unaberrated gradient of the quadrupole. The second special parameter is the phase angle α_N of the multipole. The spatial dependence of the magnetic potential giving rise to a multipole is then given by¹⁾ $r^N \sin(N\theta - \alpha_N)$. Since the multipole is normalized to the gradient it may be introduced once and allowed to apply to all succeeding quadrupoles, independent of excitation.

Coding for special parameters

1. $\epsilon(1)$ - A measure of the second-order variation with x of the magnetic field of a bending magnet. It is equal to the error in field due to this component evaluated at one horizontal unit from the beam axis, divided by the central field value, or $\Delta B_2/B_0$ at $x = 1$. Normally this parameter is set equal to zero. In order for it to produce an effect, a second-order calculation must be specified by a 17.0 card. Otherwise, it will be taken to be equal to zero even if a non-zero value is specified.
5. $g/2$ - The gap height of a bending magnet. It is used for calculating the effect of the finite extent of the fringing field of a bending magnet, but if it is non-zero, it is also taken for the vertical half-aperture of all subsequent bending magnets. Normally it is taken to be zero.
- 7-8. K_1 and K_2 - Values of dimensionless integrals describing the fringing field behavior. Normally K_1 , indicated by index code 7, is taken to be 0.5, and has a first-order effect. The quantity K_2 , indicated by index code 8, is normally zero, and is felt only in second order.
9. Z_{\max} - If a type code 16. 9. Z_{\max} is present in the beam line, DECAY TURTLE will track only those parent particles which decay in the portion of the beam line from

$$0 \leq \ell \leq Z_{\max}$$

If this element is not present, it will use the real physical decay distribution. The use of this element causes all particles to decay within the channel, thus improving the

statistics for daughter particles. The enhancement factor is printed by the program.

109. -- If a type code 16. 109. Z_{\min} is present, DECAY TURTLE will further restrict the possible decay position so that $Z_{\min} < \lambda$. This element may be used alone, or in conjunction with the 16. 9. Z_{\max} ; element.
11. p_0 - Design momentum. The type code 16. 11. p_0 ; permits the design momentum of the beam line to be redefined at any point. Its only effect is to change the radius of curvature of the reference orbit through bending magnets to correspond to a momentum p_0 .
- 12-13. $1/R1$ and $1/R2$ - The reciprocals of the radii of curvature of the entrance and exit faces, respectively, of a bending magnet. A positive value indicates the curvature is convex. Normally they are taken to be zero, meaning that the faces of the magnet are flat. They have an effect only if a second-order card, 17.0, is inserted.

Coding for magnet apertures

The magnet apertures explained below are effective only if a 13. 10. ; card is present in the beam. In decay TURTLE the 13. 10. ; card activates the apertures only for the parent particles. For the charged daughters a 13. 110. ; must be used and for the neutral daughters a 13. 210. ; card.

- 4-5. $w/2$ and $g/2$ - The horizontal and vertical half apertures, respectively, of a bending magnet. The gap height g is also

used in the calculation of the fringing-field effects. Both are normally taken to be zero. If either is non-zero and the apertures are activated, they will act as an aperture stop. A ray will be stopped if it does not go through the aperture specified.

100-102. Quadrupole apertures - Normally, if aperture constraints are turned on by a 13. 10. ; entry (or 13. 110. ; or 13. 210. ;), the quadrupoles are assumed to have circular apertures with a radius taken from the type code 5.0 cards. However, if a 16. 100. or 16. 101. type code occurs in a beam line, the circular aperture is deactivated for all subsequent quadrupoles and cannot be reactivated. Instead, the program will use the aperture specified on the 16. 100. or 16. 101. card. The type code 16. 100. a ; specifies the horizontal semi-axis of an elliptical aperture. If used alone, the aperture is taken to be circular. If it is followed by a 16. 102. b ; element, the b is taken as the vertical semi-axis of the same elliptical aperture. The type code 16. 101. a ; specifies a hyperbolic aperture, where a is the radius of a circle tangent to the hyperbola

$$|xy| \leq a^2/2 .$$

Elliptic and hyperbolic apertures may be specified simultaneously for the same magnet, but both will act as an aperture constraint only if apertures are activated by the 13. 10. ; type code.

Coding for particle charges and names

The charges and names of both parent and progeny particles may be specified. Normally, the parent and first progeny particles have unit positive charge, and the second progeny particle is neutral. The charges of any of the three particles may be respecified to be positive or negative unit charge or neutral. The code digits are:

- 200 - for parent particle
- 201 - for first progeny particle
- 202 - for second progeny particle.

The particle charge follows the code digit and must be zero or ± 1 . The particle name is inserted after the charge and must be enclosed within quotes. The name must not be longer than four characters. If a particle charge is not given, the normal option is assumed. DECAY TURTLE does not require charge conservation. If a particle name is not given, DECAY TURTLE uses the name "RAYS."

Examples:

- 16. 200 1 "PI" ;
- 16. 201 "MU" ;
- 16. 202 0. "NU" ;

SECOND-ORDER GEOMETRIC AND MAGNETIC EFFECTS: Type Code 17.0

The insertion of a type code 17.0 indicates that second-order geometric and magnetic field effects are taken into consideration. All chromatic effects discussed are taken into account whenever $\delta = \Delta p/p_0$ is not zero. Also, all quadrupole multipole moments are effective whenever specified. A 17.0 card is necessary to cause second-order geometric transfer matrix elements or second-order field variations in a bending magnet to have an effect. It is also necessary in order for the magnetic field of a sextupole to have an effect. Without a 17.0 card a sextupole is taken to be a drift space.

In decay TURTLE, a second-order calculation is automatically invoked when a type code 60.0 card is present in the data deck calling for a decay calculation.

SEXTUPOLE: Type Code 18.0

A sextupole may be inserted for correction of second-order aberrations. Unless a 17.0 card is included it acts like a drift space.

Four parameters are needed:

- 1 - Type code 18.0.
- 2 - The effective length ℓ of the field
(normal unit is metres).
- 3 - The pole tip field B_0 (normal unit is kG).
- 4 - The pole tip half aperture a . The effective strength of the sextupole is $qB_0\ell/a^2p$.

The first- and second-order transformation elements for a sextupole are given in Ref. 1. In TURTLE these elements are evaluated using the actual momentum of the ray. Therefore, TURTLE can give a very good indication of how effectively chromatic aberrations can be eliminated or minimized in a beam line.

SOLENOID: Type Code 19.0

The solenoid is most often used as a focusing element in systems passing low-energy particles. Particles in a solenoidal field travel along helical trajectories. The solenoid fringing-field effects necessary to produce the focusing are included.

There are three parameters:

- 1 - Type code 19.0.
- 2 - Effective length ℓ of the solenoid (meters).
- 3 - The field (kG). A positive field by convention points in the direction of positive z for positively-charged particles.

BEAM ROTATION: Type Code 20.0

A rotation of the coordinates of a ray at a given point in the beam line about the beam axis may be effected by a 20.0 type code. Thus one can simulate a magnet oriented differently than is normally available, or a known rotational misalignment of an element about the beam axis. There are two parameters:

- 1 - Type code 20.0.
- 2 - The angle of rotation in degrees.

The rotation is taken in a clockwise sense about the beam axis.

DEFINED SECTION: Type Code 24.0

A system may contain a section which is repeated at some later point. The repeated section may not occur immediately after the original section, so that the repeat element (type code 9.0) is not appropriate. It may also be useful to repeat a section, but with the elements listed in the reverse order.

The 24.0 type code element allows definition of the section to be repeated. It also indicates the locations at which the section is to be repeated and whether the repetition is to be forwards or backwards.

Three parameters are required:

- 1 - Type code 24.0.
- 2 - Code digit.
- 3 - Section name. The section name is a maximum of four characters long and is enclosed in single quotes.

The meaning of the code digit is as follows:

- 1 - Indicates the beginning of the section to be defined.
- 2 - Indicates the end of the section to be defined.
- 3 - The section named is to be repeated at the present location in the forwards direction.
- 4 - The section named is to be repeated at the present location with the elements in reverse order.

A number of rules apply to the use of the defined section element. Each defined section must have both its beginning and its end indicated, and the beginning must precede the end. The definition of a section must completely precede its use. A given name can be used only once to define a section, although it can be used many times to indicate a repeat of a section. A defined section may not refer to itself, either explicitly or implicitly. The number of defined sections is limited to 10. The total z rotation (type code 20.0) within a defined section must sum to zero. Finally, defined sections must nest properly with the repeat code (type code 9.0). Defined sections, however, need not nest properly with each other.

Example of a defined section

24.	1.	'DOUB'	;	_____	Begin defined section.
5.	10.	5.	2.	;	
3.	8.	;			
5.	10.	-5.	2.	;	
24.	2.	'DOUB'	;	_____	End defined section.
24.	3.	'DOUB'	;	_____	Repeat defined section.
24.	4.	'DOUB'	;	_____	Repeat defined section
					in reverse order.

HISTOGRAMS: Type Codes 50.0, 51.0 and 52.0

A one- or two-dimensional histogram of beam parameters (listed below) may be requested at any position in the beam line. A one-dimensional histogram is indicated by a 50.0 card. The horizontal coordinate of a two-dimensional histogram is indicated by a 51.0 card and the vertical coordinate by a 52.0 card. The histogram card is placed at the point in the beam line where one wishes to histogram the ray coordinate specified. Any of these three type codes requires five parameters:

- 1 - Type code 50.n, 51.0 or 52.n, indicating the type of histogram desired. In DECAY TURTLE, 51.n is also a valid entry.
- 2 - A code number for the parameter to be histogrammed (see below).
- 3 - The lower limit of the histogram.
- 4 - The upper limit of the histogram.
- 5 - The interval of the histogram (bin size).

Normally the symbol n is blank or zero. If n is a digit in the range 1 to 9, the histogram is said to be flagged. An entry is made into such a histogram only for rays which have encountered a "flag" element of the form 53.n; before or after the histogram specification, where n has the same value as on the histogram specification card.

The meaningful coordinates for the histograms are specified by the following codes:

- 1 - x coordinate
- 2 - x' coordinate
- 3 - y coordinate
- 4 - y' coordinate
- 6 - $\delta = (p - p_0)/p_0$.

In DECAY TURTLE, eight more codes are recognized:

- 8 - z, the distance along the reference orbit. Such a histogram will show the distribution of particles which have been stopped by aperture limits.
- 11 - p, the momentum of the particles.
- 12 - COST, the cosine of the center-of-mass decay angle of the parent particle.
- 18 - ZD, the distance of the decay point along the reference orbit.
- 21 - $r = \sqrt{x^2 + y^2}$, the radius.
- 22 - $XI = \sqrt{x'^2 + y'^2}$, the emittance angle, the angle the ray makes with respect to the central trajectory.
- 23 - PSI = arctan (y/x), the polar angle.
- 24 - ZETA = arctan (y'/x').

The codes given above all apply to the parent particles only. In order to specify the charged daughter particles, add 100 to the relevant code. For the neutral daughters, add 200 to the relevant code.

If either of the coordinates specified for a two-dimensional histogram is the distance along the reference orbit (code 8.0), the histogram will

refer only to particles lost from the beam. Such a loss histogram cannot be flagged. A maximum of 10 such histogram is allowed.

A one-dimensional histogram will be created at any point in a beam line where a 50.0 type code card is inserted. The intervals of the histogram are arranged vertically and specified on the left side of the page. The beam intensity in a given interval is indicated by a horizontal row of the letter "X".

A two-dimensional histogram is actually initiated by the 52.0 card, while the 51.0 card merely supplies a value for the horizontal coordinate. Thus the 51.0 card and the 52.0 card for a two-dimensional histogram need not occur at the same location in the beam line. However, for each 52.0 card there must be one 51.0 card somewhere in the deck preceding it. Several 52.0 cards can use the same 51.0 card to supply the horizontal coordinate for the histogram. The 51.0 card must simply occur at some point in the deck earlier than the first 52.0 card. Whenever a 52.0 card appears, indicating a vertical coordinate and initiating a histogram, the horizontal coordinate is taken from the 51.0 card most recently preceding it.

For both one- and two-dimensional histograms the coordinates histogrammed with their units and position in the beam line are printed with the histogram. Examples of both one- and two-dimensional histograms are given below.

For a two-dimensional histogram the number of rays falling in a given bin is printed directly for 1-9 rays, represented by a letter A-Z for 10-35 rays, and represented by a \$ for more than 35 rays. The \$ was chosen because it is a fairly dark symbol. Sums of the rows and columns are given along the edges of the histogram. The numbers giving the sums of columns are themselves to be read vertically. Overflow in all four directions is given below the histogram.

A total of 100 histograms in a given beam line is allowed. The number of intervals allowed per histogram is limited directly only for the horizontal coordinate of a two-dimensional histogram, the limit being 100. If the limits and interval size specified are such that more than 100 intervals would result, the program readjusts the upper limit so that the number of intervals equals 100. There is a limit of 10,000 total locations provided for histogram storage. If N is the number of intervals used in a one-dimensional histogram, the number of storage locations used is $N+3$. If N_A and N_D are the number of intervals used for the horizontal and vertical coordinates respectively of a two-dimensional histogram, the number of locations used is $(N_A+1) \cdot (N_D+1)+5$.

HISTOGRAM FLAG: Type Code 53.n

If an integer is placed after the decimal point on type codes 50.0, 51.0, or 52.0 (i.e., 50.3 or 52.5, for example), then that histogram is flagged. Rays will not be entered in it unless they reach a location where there is a 53.0 element with the same number after the decimal. Unflagged histograms are not affected. For original TURTLE, the two coordinates of a two-dimensional histogram can refer to only one kind of particle. As such, only one flag can be associated with a two-dimensional histogram and must be specified on the 52.n element. Any flag on the 51.n element will be ignored. In DECAY TURTLE, however, a flag is permitted on both cards, and both flags must be satisfied for the histogram to be entered.

There is but one parameter required:

- 1 - Type code 53.n.

If just this one parameter is used, then the required particle type for the flag is taken from the histogram specification which refers to the flag. Thus, if two histogram specifications use the same flag, but refer to two different particle types, then the required particle for the flag will depend on the histogram which is flagged. The required particle for a flag to be satisfied can also be specified explicitly on the flag card. In this case, the particle types for the flagged histogram and the flag itself may be different. For example, one can histogram all pions which subsequently decay and produce muons passing a later point. The second parameter, specifying particle type, is zero for the parent particle, 100 for the charged daughter, and 200 for the

neutral daughter. An explicit muon flag, for example might be:

53.3 100. ;

In addition, any aperture constraint may be required for a flag to be satisfied. The aperture types are the same as for type code 6. The number giving the aperture type is added to the second parameter. A third parameter gives the half width of the aperture. An elliptic aperture is also available as described under type code 6. The aperture constraint does not intercept the trajectory, but is required for the flag to be satisfied. Continuing the previous example, a flag requiring a muon to pass with x less than .4 may be written as:

53.3 101. .4 ;

Obviously, if an aperture condition is placed on a flag, the particle type must be specified explicitly, since the second parameter is present.

Example No. 1

Histogram of the phase-space acceptance and emittance of a beam line:

1. ; Initial beam specification.

51.	1.	-20.	20.	1.	;	} Histogram for x,x' phase-space acceptance passing through beam line and reaching flagged
52.3	2.	-10.	10.	.5	;	

position (53.3) at end of beam.

.	}	Element cards specifying beam line.
.		
.		

53.3 Flag for acceptance histogram (above).

51.	1.	-20.	20.	1.	;	} Histogram for x,x' phase-space (emittance) at end of beam line.
52.	2.	-10.	10.	.5	;	

SENTINEL.

Example No. 2

Histograms for a typical decay channel:

1. ; Initial phase-space (BEAM) specification.
- 51.1 6. -10. 10. .5 ; The $\delta = \Delta p/p_0$ of parents at this
. location is taken as the horizontal
. coordinate for the histogram to be
. displayed at the location of the
. 52.0 card.
. .
- 53.1 Only those parents that have passed through this position
. are used for the histogram.
. .
- 53.2 Only those neutral daughters that were generated before
. this position (i.e., those passing through this position)
. and which reach the position of the 52.2 card are entered
. into the histogram.
. .
- 52.2 211. 0. 40. 1. ; The momentum p of the neutral
. daughter is used as the vertical
. coordinate for the histogram
. displayed at this location.

Since an entry is made in the histogram only if both particles are present, the result is a histogram containing only those particles involved in the decay process between the two flagged positions 53.1 and 53.2. No other 51.0 type code entry should appear between the 51.1 and 52.1 entries, otherwise the flagging process will not function as described above.

Examples of a one-dimensional and a two-dimensional histogram are given in Figs. 3 and 4.

HORIZONTAL AXIS X IN MM 1003.806 FT FROM THE TARGET
 VERTICAL AXIS Y IN MM 1003.806 FT FROM THE TARGET

	-0.006	0.000	0.006	TOTALS
	I**-----**-----**I-----			
-0.006 TO -0.005	I	11	I	2
-0.005 TO -0.005	I	21 11 1	11 I	8
-0.005 TO -0.004	I	11 21 1 1	I	7
-0.004 TO -0.004	I	3562214 1 2	I	26
-0.004 TO -0.003	I	1 12436A9773112	I	57
-0.003 TO -0.002	I	11346HBONIE71 11	I	132
-0.002 TO -0.002	I	554GHXV\$\$\$SMA4 11 1	I	274
-0.002 TO -0.001	I	568JX\$\$\$\$\$\$JD68121	I	526
-0.001 TO -0.001	I	22387U\$\$\$\$\$\$MO5313	I	671
-0.001 TO 0.000	I	1377GR\$\$\$\$\$\$H94 1	I	825
0.000 TO 0.001	I	234DEN\$\$\$\$\$\$O7311	I	838
0.001 TO 0.001	I	3118EU\$\$\$\$\$\$ULD2 1	I	706
0.001 TO 0.002	I	11349SMT\$\$\$\$\$NM953	I	454
0.002 TO 0.002	I	14437FNQT\$UJB62 2	I	221
0.002 TO 0.003	I	2359EFJI8E412	I	114
0.003 TO 0.004	I	1 111 13AA79243 1 1	I	55
0.004 TO 0.004	I	2 3213523111 1	I	25
0.004 TO 0.005	I	2 13 4 11 1 1	I	14
0.005 TO 0.005	I	1 12 2	I	6
0.005 TO 0.006	I	1 1 1	I	3
	I**-----**-----**I-----			
	I		I	
	I		I	
	I	1356776421	I	
	I	1136882349901145311	I	
TOTALS	I	12528240180909060133	I	4964

TOTAL NUMBER OF ENTRIES = 5000 INCLUDING UNDERFLOW AND OVERFLOW AS FOLLOWS

	UNDERFLOW	OVERFLOW
ACROSS	18	4
DOWN	6	8

HORIZONTAL AXIS X IN MM 1003.806 FT FROM THE TARGET
 VERTICAL AXIS Y IN MM 1003.806 FT FROM THE TARGET

Fig. 4. Example of a two-dimensional histogram of X versus Y.

TEMPORARY STORAGE OF VALUES FOR HISTOGRAMS: Type Code 54.0

Occasions may arise when it is desirable to produce a two-dimensional histogram where the horizontally displayed coordinate is something other than the one appearing on the most recent 51.0 card. Also, one may wish to display horizontally the coordinate encountered further downstream. Trajectory coordinates may now be stored temporarily to allow such possibilities. Ten registers are available for storage. These parameters are required to place a trajectory coordinate in one of the registers:

- 1 - Type code 54.0.
- 2 - Coordinate number. The numbers have the same meaning as on the histogram cards. (Type codes 50.0, 51.0 and 52.0).
- 3 - Register number. The possibilities are 1 through 10.

A histogram card may specify a coordinate in a stored register by having the coordinate number on the histogram card be 50 plus the register number. Thus

50. 53. -1. 1. .1 ;

produces a one-dimensional display of the contents of register three. No particle type is specified on a 54.0 element, but is taken from the histogram specification.

A user should bear in mind that a histogram is not entered until all the associated histogram specification cards have been encountered. For example, both coordinates may be placed in storage at intermediate points

in the beam line, while the histogram specification may be placed at the end. In this case, no entry will be made in the histogram unless either the parent or a decay product reaches the end. For this reason, it is recommended that the 54.0 element be used only as necessary, and not for convenience in placing all histogram requests together.

DECAY: Type Code 60.0

Normally DECAY TURTLE will track parent particles as in the original TURTLE. If decay calculations are to be included, an element with five parameters must be inserted among the data cards, normally after the BEAM cards:

- 1 - The type code 60.0 specifying decay.
- 2 - The mass of parent particles (GeV).
- 3 - The mass of charged daughter particles (GeV).
- 4 - The mass of neutral daughter particles (GeV).
- 5 - The parent lifetime in nanoseconds.

The particle masses are given in the same units as the design momentum of the beam line (ignoring the factor c , normally GeV). The unit for mass can be changed by means of a 15. 11. type code entry preceding the 60.0 type code card.

The charges of the parent and two daughter particles may be respecified via 16. 200. through 16. 202. elements. However, in this manual, we continue to refer to the two progeny particles as the charged daughter and neutral daughter respectively. It is even possible to respecify the "charged daughter" to be, in fact, neutral, and the "neutral daughter" to be charged.

The standard version of decay TURTLE contains the decay kinematics for isotropic two-body decay. The particle masses must always be specified; at present no default values are known to the program. A different decay process can be accommodated by changing one or more of the following subroutines:

SUBROUTINE DECAY

generates from the parent RAY the two daughter rays:

RAYC for the charged daughter,

RAYN for the neutral daughter.

SUBROUTINE DECAYP

decodes the 60.0 type code card and prepares data for DECAY.

SUBROUTINE DECAYZ(ZD)

returns the generated random decay position.

The initial phase space of the parent beam has been described in the section on type code 1.0. For decay problems, it is often required to enter a different initial distribution. In such a case the user has to change the SUBROUTINE EMIT which generates a new parent ray at each call.

Users who wish to alter one of the indicated routines are referred to the comments in the main program which describe the significance of all COMMON variables used in the program.

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